Digital Image Processing

Morphological Image Processing (2)

Topics

- Morphological Operations
  - Connected Component Extraction
  - Convex Hull
  - Thinning
  - Thickening
  - Skeleton
  - Pruning
  - Extension to gray level images
  - Matlab Examples

Dilation and Erosion

- Dilation and Erosion are two basic operations in morphological processing.
- Dilation of a set A in Z^2 by a set B in Z^2 is denoted by $A \oplus B$ and given by

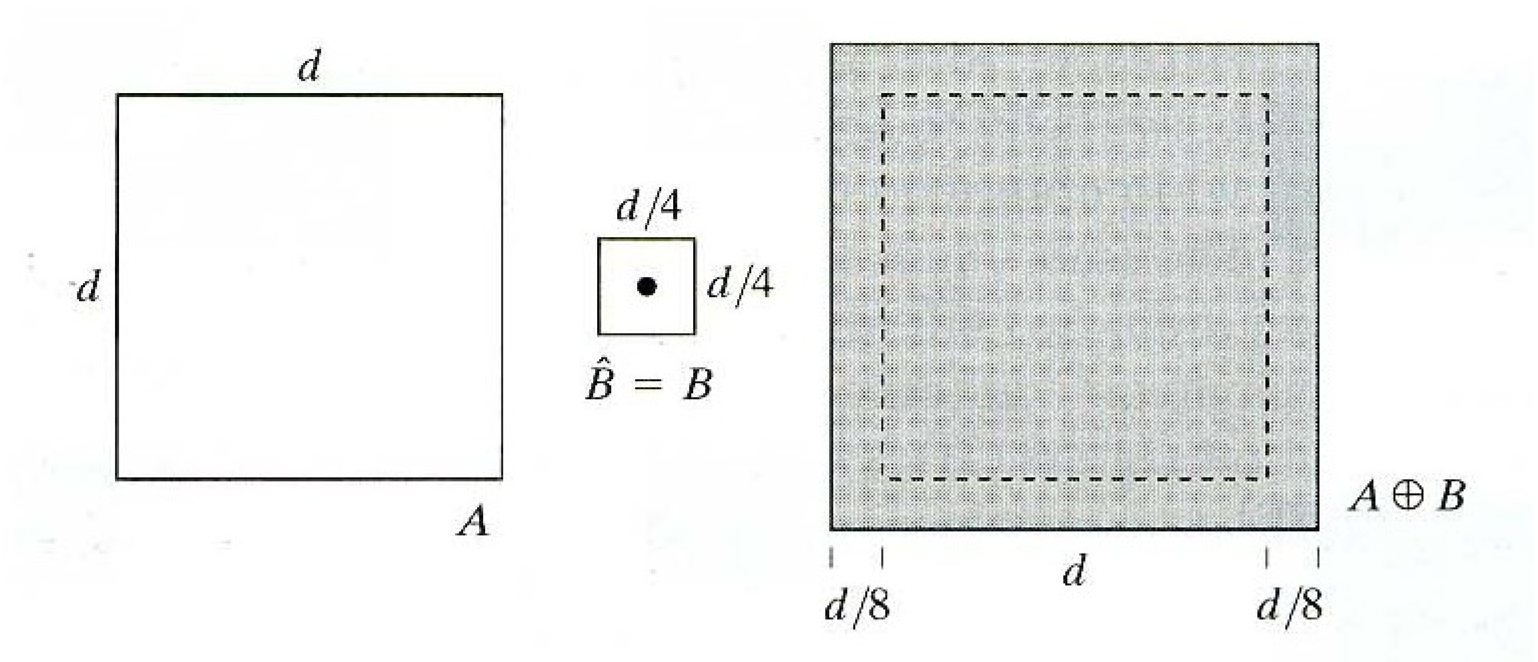
$$A \oplus B = \left\{ z \mid \left(\hat{B} \right)_z \cap A \neq \varnothing \right\}$$

Dilation

- The dilation of A by B is the set of all displacements such that A and $\hat{B}$ overlap with at least one point

- B is called structuring element

Dilation



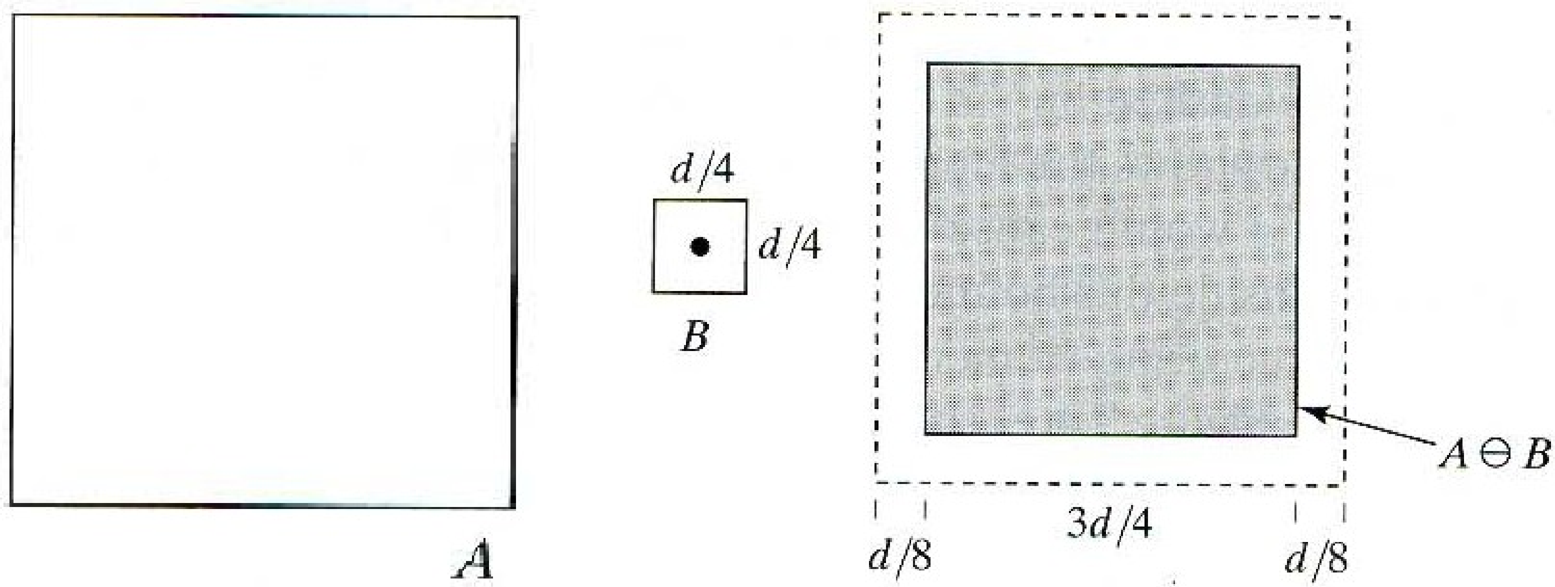
Erosion

- Erosion of a set A in Z^2 by a set B in Z^2 is denoted by $A \ominus B$ and given by:

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}$$

Erosion of A by B is the set of all points z such that B translated by z is contained in A

Erosion



Opening

- Opening smoothes the outer contours, breaks narrow connections, and eliminates small protrusions.

- Opening is defined as :

$$A \circ B = (A \ominus B) \oplus B$$

Closing

- Closing smoothes the object contour, fuses narrow connections, eliminates small holes and gaps.

$$A \bullet B = (A \oplus B) \ominus B$$

Extraction of Connected Components

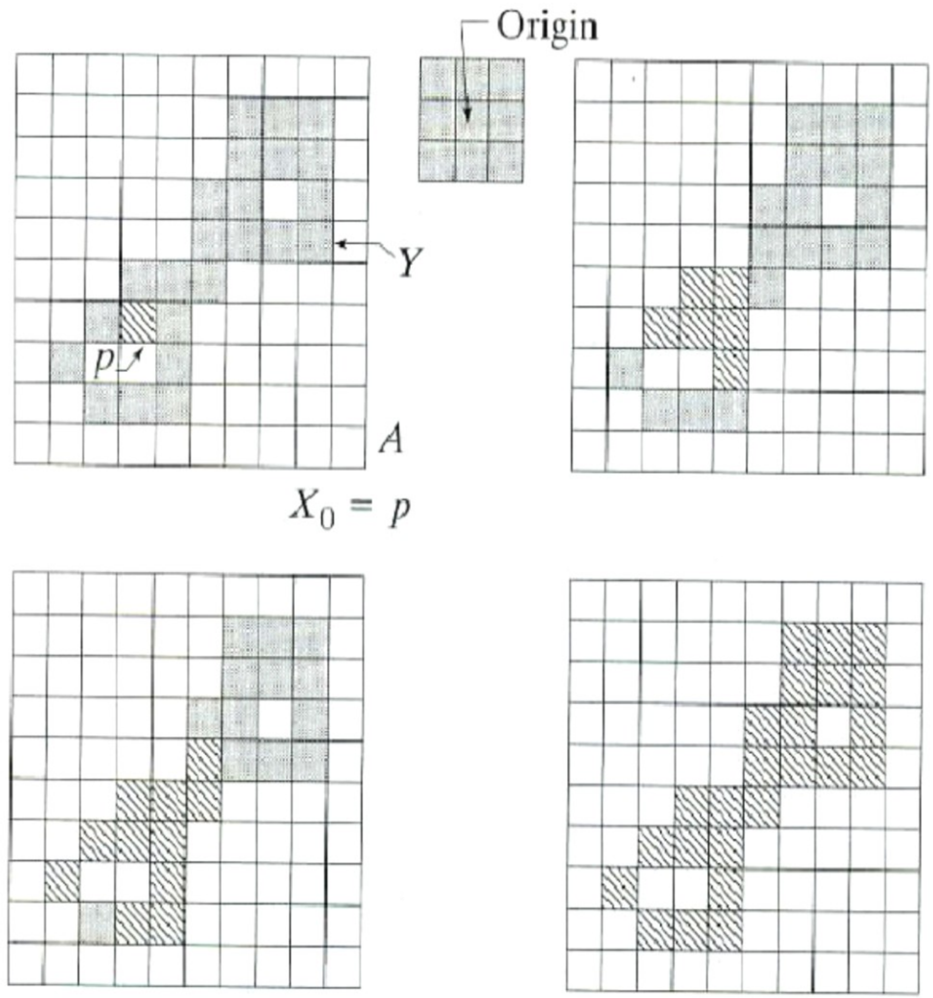
- Begin with a point P inside the connected component, iterate:

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \ldots$$

Until $X_k = X_{k-1}$

Initially $X_0 = P$

Connected Component Extraction

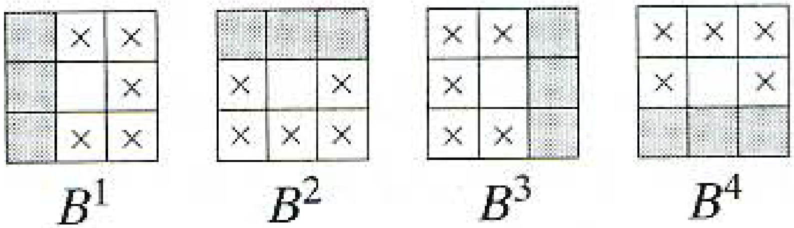


Convex Hull

- A set is said convex if the straight line connecting any two points of the set lies entirely within A.
- Convex Hull of set S is the smallest convex set A that contains S
- The set difference A-S is called the convex deficiency of S

Computing Convex Hull

- Let B_i for i=1,2,3,4 represent the structuring elements shown below



Convex Hull

- Repeat the following equation until converge

$$X_k^i = (X_{k-1} \circledast B^i) \cup A \quad i = 1, 2, 3, 4 \quad \text{and} \quad k = 1, 2, 3, \ldots$$

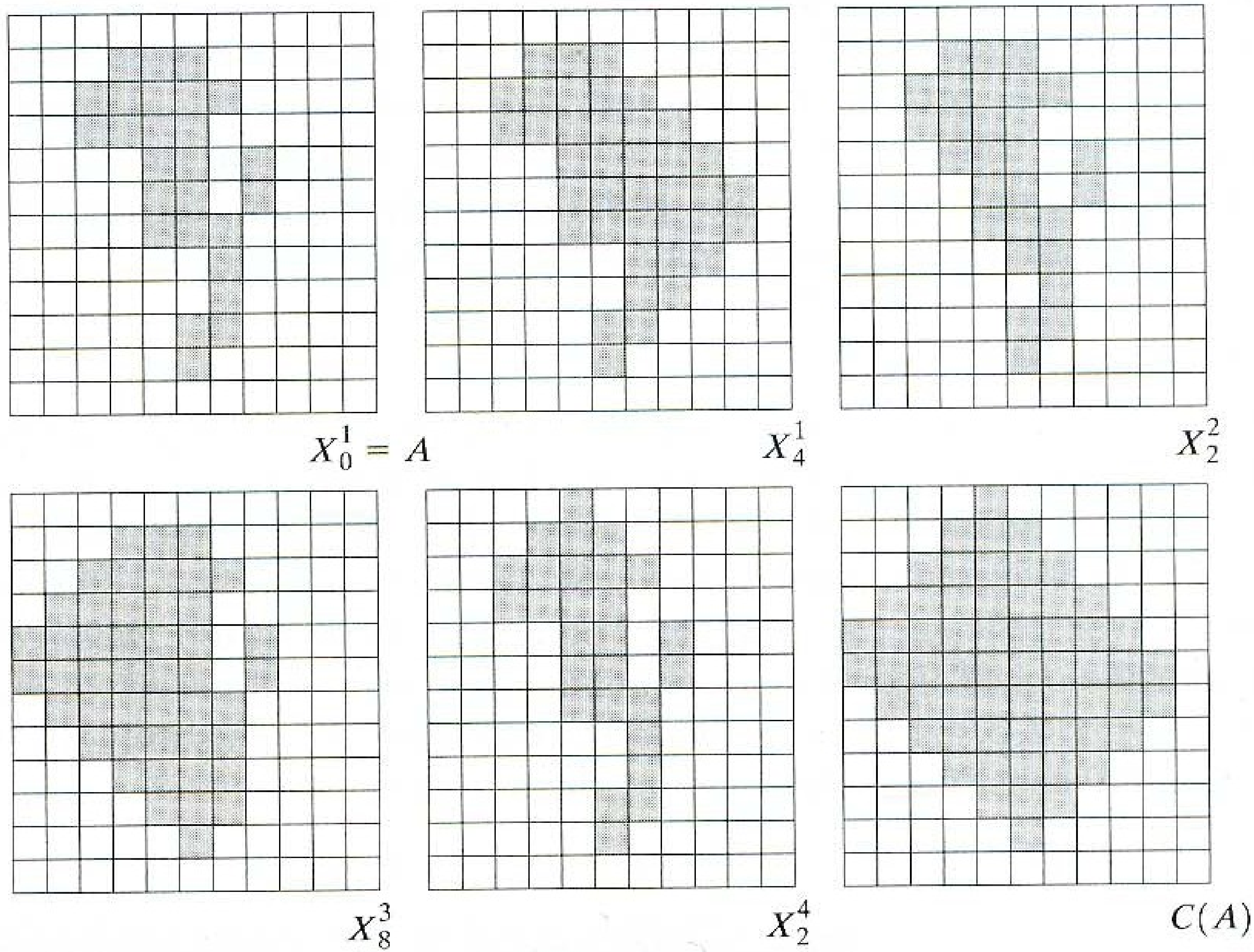
with

$$X_0^i = A$$

$\circledast$ is the Hit-or-Miss operator

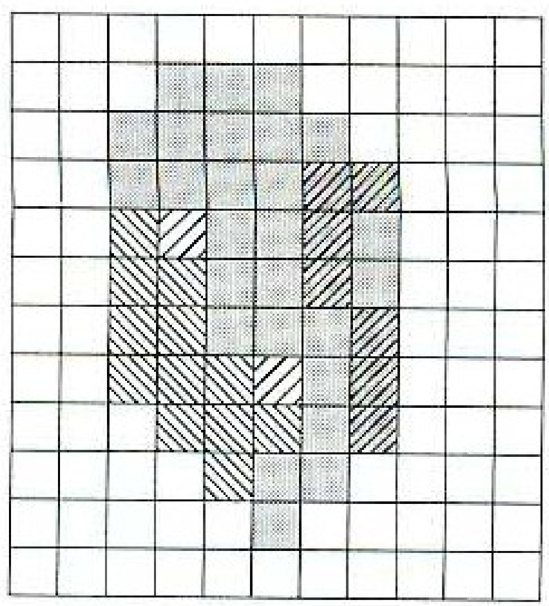
Assuming $D^i = X_{\text{conv}}^i$ Convex Hull is $C(A) = \bigcup_{i=1}^{4} D^i$

Example (Convex Hull)



Improving Convex Hull Algorithm

- The algorithm can be improved by limiting the growth of the algorithm beyond the maximum dimensions of the original set.



Thinning and Thickening

- Thinning is an image-processing operation in which binary valued image regions are reduced to lines
- The purpose of thinning is to reduce the image components to their essential information for further analysis and recognition
- Thickening is changing a pixel from 1 to 0 if any neighbors of the pixel are 1.
- Thickening followed by thinning can be used for filling undesirable holes.
- Thinning followed by thickening is used for determining isolated components and clusters.

Thinning

- Thinning is defined in terms of hit or miss as

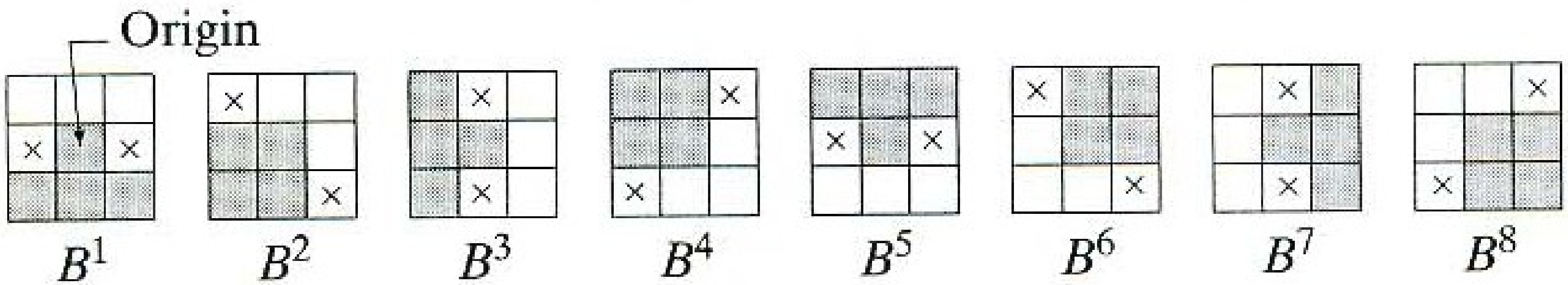
$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

where B is a sequence of structuring elements like
$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$ and the operation can be given as

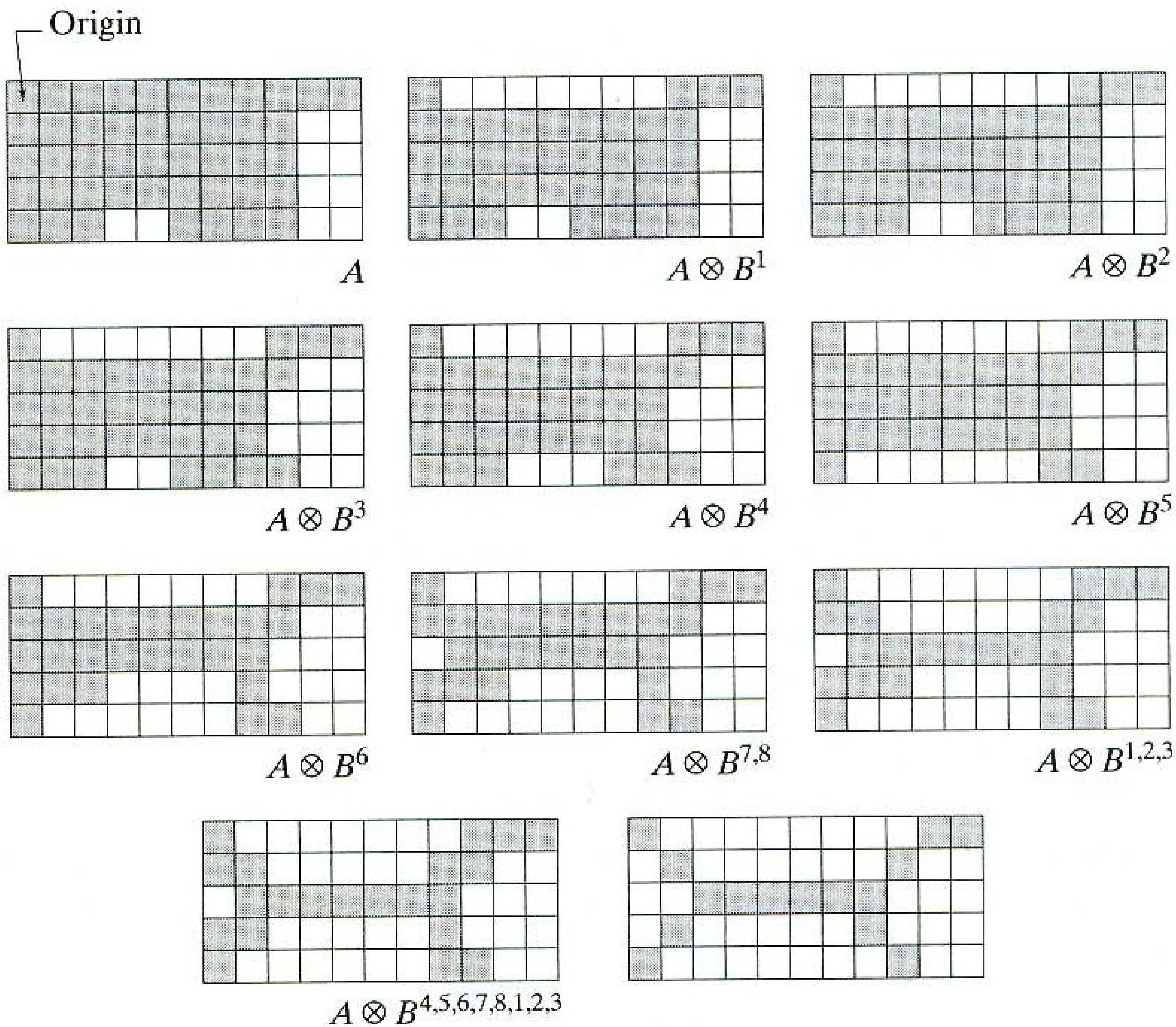
$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$$

Thinning

- Sample set of structuring elements



Thinning Example



Thickening

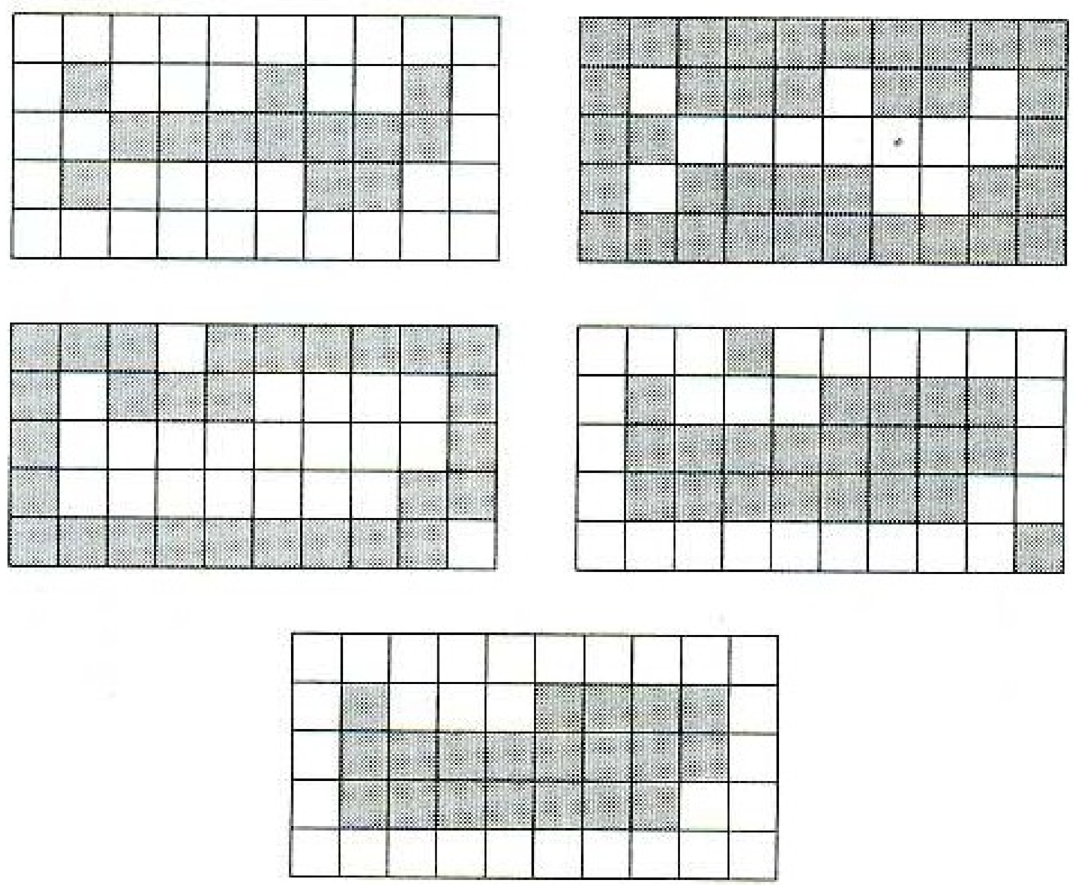
- Thickening is the morphological dual of thinning and defined as

$$A \odot B = A \cup (A \circledast B)$$

or

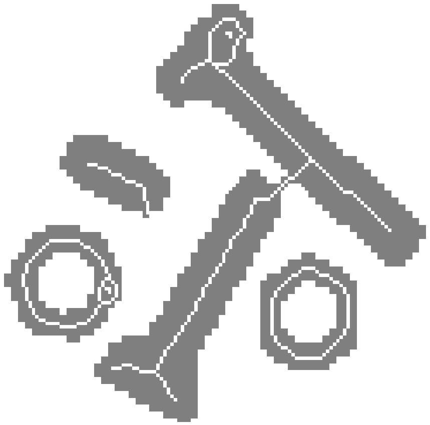
$$A \odot \{B\} = ((\dots((A \odot B^1) \odot B^2)\dots) \odot B^n)$$

Thickening Example



Skeleton

- The informal definition of a skeleton is a line representation of an object that is:
  - one-pixel thick,
  - through the "middle" of the object, and,
  - preserves the topology of the object.



Skeleton

- Skeleton is defined by

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

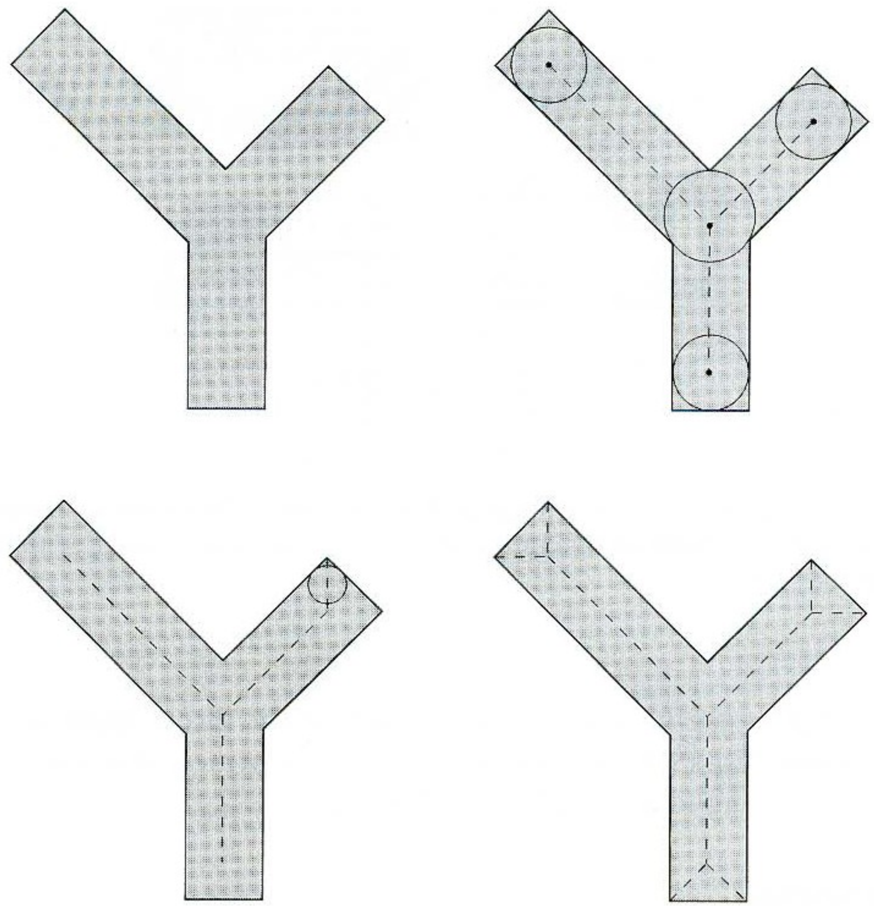
where

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

k is the last iterative step before A erodes to an empty set

$$K = \max\{k \mid (A \ominus kB) \neq \varnothing\}$$

Skeleton Example

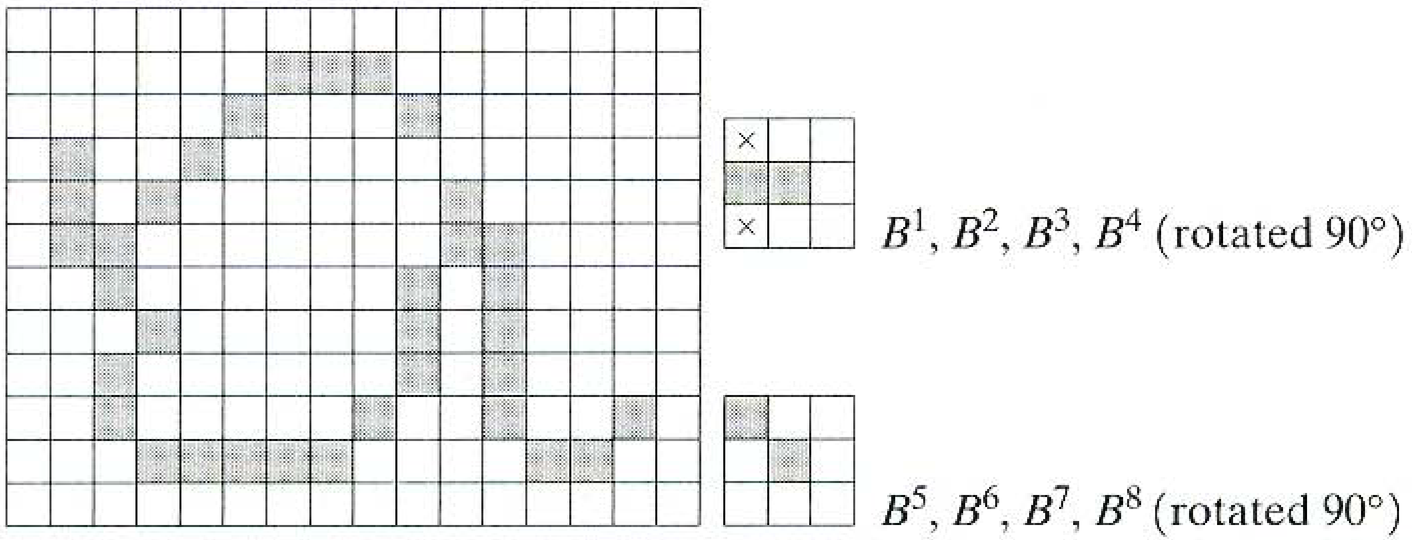


Pruning

- Thinning and skeletonizing algorithms need a clean-up post-processing
- The following steps are used for pruning:
  - Thinning $$X_1 = A \otimes \{B\}$$
  - Find the end points $$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$
  - Dilate end points $$X_3 = (X_2 \oplus H) \cap A$$
  - Find the union of X1 and X3 $$X_4 = X_1 \cup X_3$$

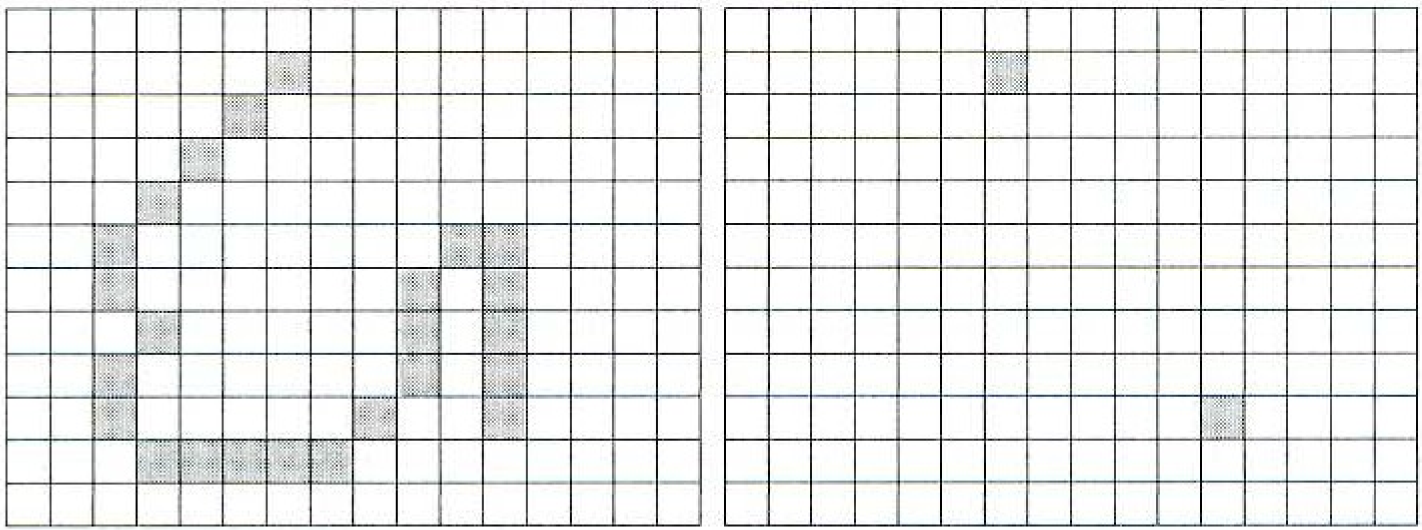
Pruning Example

- Original image and structuring elements



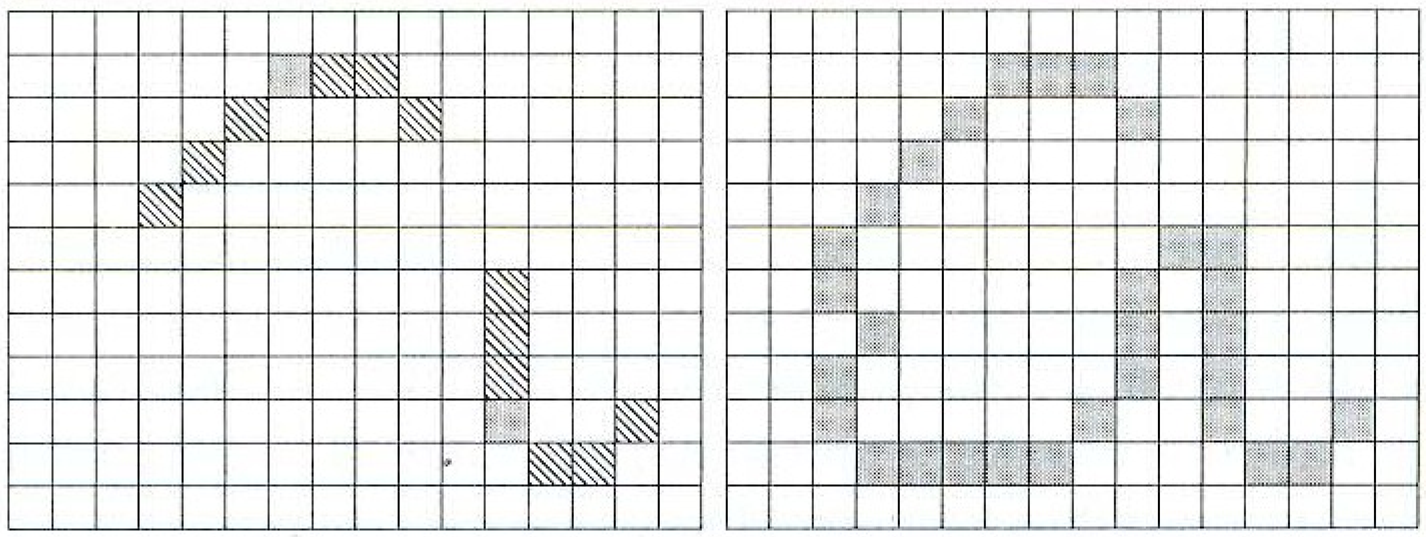
Pruning Example

- Result of thinning and end points detected



Pruning Example

- Dilation of end points and the pruned image



Extension to Gray Level

- Dilation is expressed in 1D as

$$(f \oplus b)(s) = \max\{f(s - x) + b(x) \mid (s - x) \in D_f \quad \text{and} \quad x \in D_b\}$$

- Erosion is given by

$$(f \ominus b)(s) = \min\{f(s + x) - b(x) \mid (s + x) \in D_f \ \text{and} \ x \in D_b\}$$

Extension to Gray Level (2D Case)

- Dilation

$$(f \oplus b)(s,t) = \max\{f(s-x, t-y) + b(x,y) \mid (s-x),(t-y) \in D_f; (x,y) \in D_b\}$$

- Erosion

$$(f \ominus b)(s,t) = \min\{f(s+x, t+y) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$

Morphological Operations in MATLAB

- To create structuring element use strel(.)

  SE = strel(*shape*, parameters)

Examples:

SE = strel('arbitrary', NHOOD)
SE = strel('diamond', R)
SE = strel('disk', R, N)
SE = strel('line', LEN, DEG)
SE = strel('octagon', R)
SE = strel('pair', OFFSET)
SE = strel('periodicline', P, V)
SE = strel('rectangle', MN)
SE = strel('square', W)

Morphological Operations in MATLAB

- SE=strel(NHOOD) is also a valid call for the function
- Use imerode(Im,SE) and imdialte(Im,SE) for erosion and dilation respectively
- Use imopen(Im,SE) and imcolose(Im,SE) for openning and closing
- For hit-or-miss use bwhitmiss(.)
  - BW2 = bwhitmiss(BW1,SE1,SE2)
    BW2 = bwhitmiss(BW1,INTERVAL)

Hit or Miss Example

```
bw = [0 0 0 0 0 0
      0 0 1 1 0 0
      0 1 1 1 1 0
      0 1 1 1 1 0
      0 0 1 1 0 0
      0 0 1 0 0 0]

interval = [0 -1 -1
            1  1 -1
            0  1  0];

bw2 = bwhitmiss(bw,interval)

bw2 =

     0     0     0     0     0     0
     0     0     0     1     0     0
     0     0     0     0     1     0
     0     0     0     0     0     0
     0     0     0     0     0     0
     0     0     0     0     0     0
```

Questions?