Digital Image Processing

Generalized Hough Transform

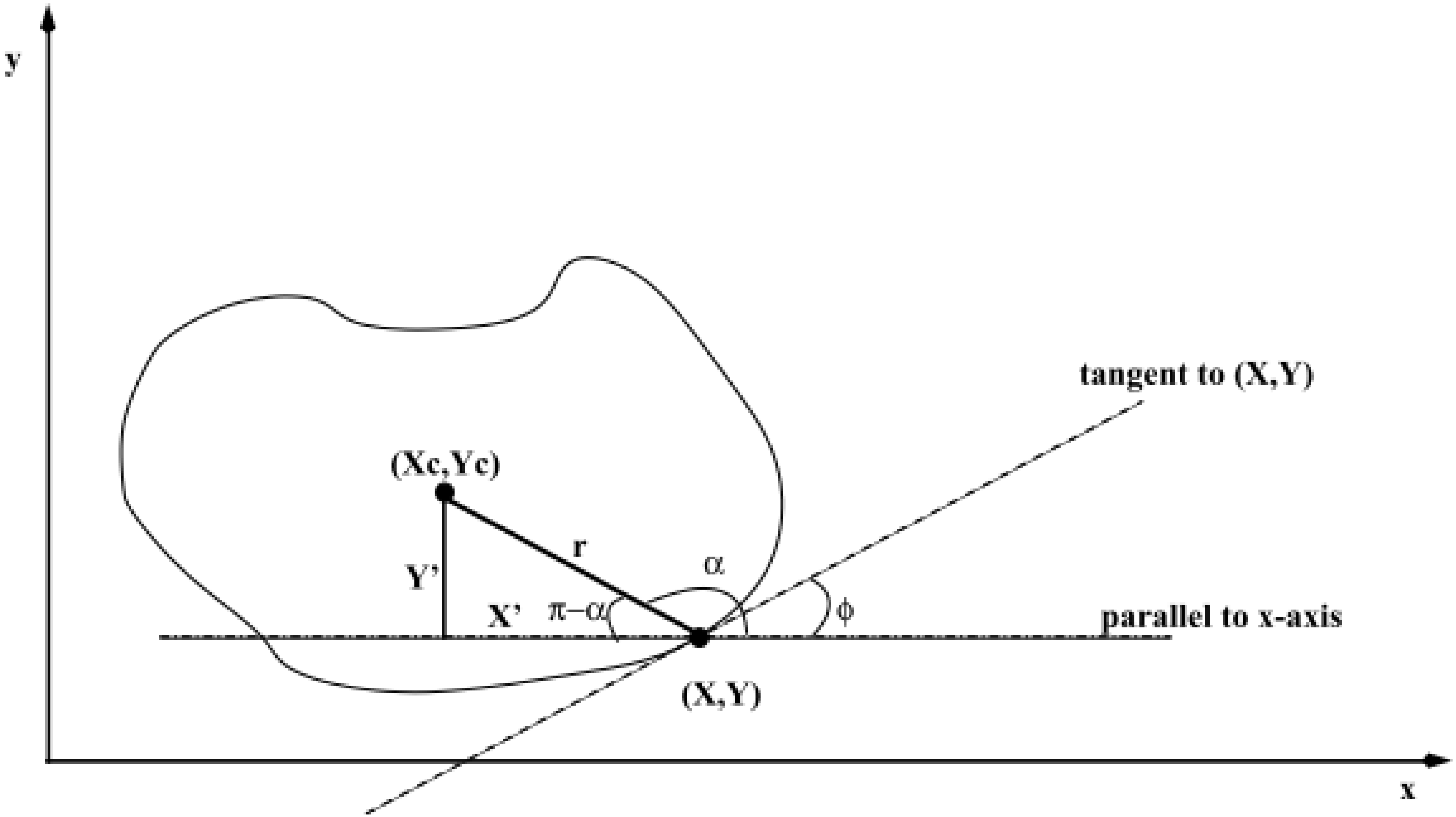
Morphological Image Processing

Introduction

- Hough Transform is used for detecting regular shapes which can be described in a parametric form.
- Example shapes are
  - Lines
  - Circles
  - Ellipses
- Question: Can we extend the method to detect more general shapes?

Shape Description

- Define the shape as follows:
  - Choose a center for the shape (Xc, Yc)
  - Create a table named R table which has a row for each angle Φ (discretized properly).
  - Connect each point on the boundary of the object to the shape center. This gives *r* (distance), a (angle) for each pair.
  - Enter each pair (r, a) into table R at row given by the angle of the tangent at that point to the shape



y
tangent to (X,Y)
(Xc,Yc)
r
α
Y'
X'
π−α
φ
parallel to x-axis
(X,Y)
x

Detection

- Shape can be detected assuming no scale change or rotation.
- We may include scale change and rotation by considering new parameters

Case 1: Fixed Scale and Orientation

- Quantize the parameter space:
  - $C[min_x \ldots max_x][min_y \ldots max_y]$
- for each edge point (x, y) do
  - Find the tangent line angle find all *(r, α)* pairs
  - Increment the value of the entry in matrix C pointed to by *(r, α)*
  - *Use*
    - $x_c = x + r\cos(\alpha)$
    - $y_c = y + r\sin(\alpha)$
  - Maximum values of C give the probable locations of the shape

Case 2: With Scale and Rotation

- Suppose rotation θ and uniform scaling ***s*** are applied to the shape:
- We have:
  - $> \quad (x', y') --> (x'', y'')$
  - $> \quad x'' = (x' \cos(\theta) - y' \sin(\theta))s$
  - $> \quad y'' = (x' \sin(\theta) + y' \cos(\theta))s$

- Replacing x' by x'' and y' by y'' we have:
  - > $x_c = x - x''$ or $x_c = x - (x' \cos(\theta) - y' \sin(\theta))s$
  - > $y_c = y - y''$ or $y_c = y - (x' \sin(\theta) + y' \cos(\theta))s$

- Quantize the parameter space:
  - $C[min_x \ldots max_x][min_y \ldots max_y][min_\Theta \ldots max_\Theta][min_s \ldots max_s]$
- for each edge point (x, y) do
  - Find the tangent line angle, and all corresponding *(r, α)* pairs
  - Increment the value of the entry in matrix C pointed to by *(r, α)*
  - *Use*
    - $x_c = x' + r\cos(\alpha)$
    - $y_c = y' + r\sin(\alpha)$

    $for(\ t = min_\Theta \ \ upto\ max_\Theta\ )$

    $\quad for(s=min_S\ upto\ max_S)$

    $\qquad x_c = x - (x' \cos(t) - y' \sin(t))s$

    $\qquad y_c = y - (x' \sin(t) + y' \cos(t))s$

    $\qquad ++(C[xc][yc][t][s]);$
  - Maximum values of C give the probable locations of the shape

Morphological Image Processing Topics

- Basic Concepts from Set Theory
- Dilation and Erosion
- Opening and Closing
- The Hit-or-Miss Transformation
- Some Basic Morphological Algorithms

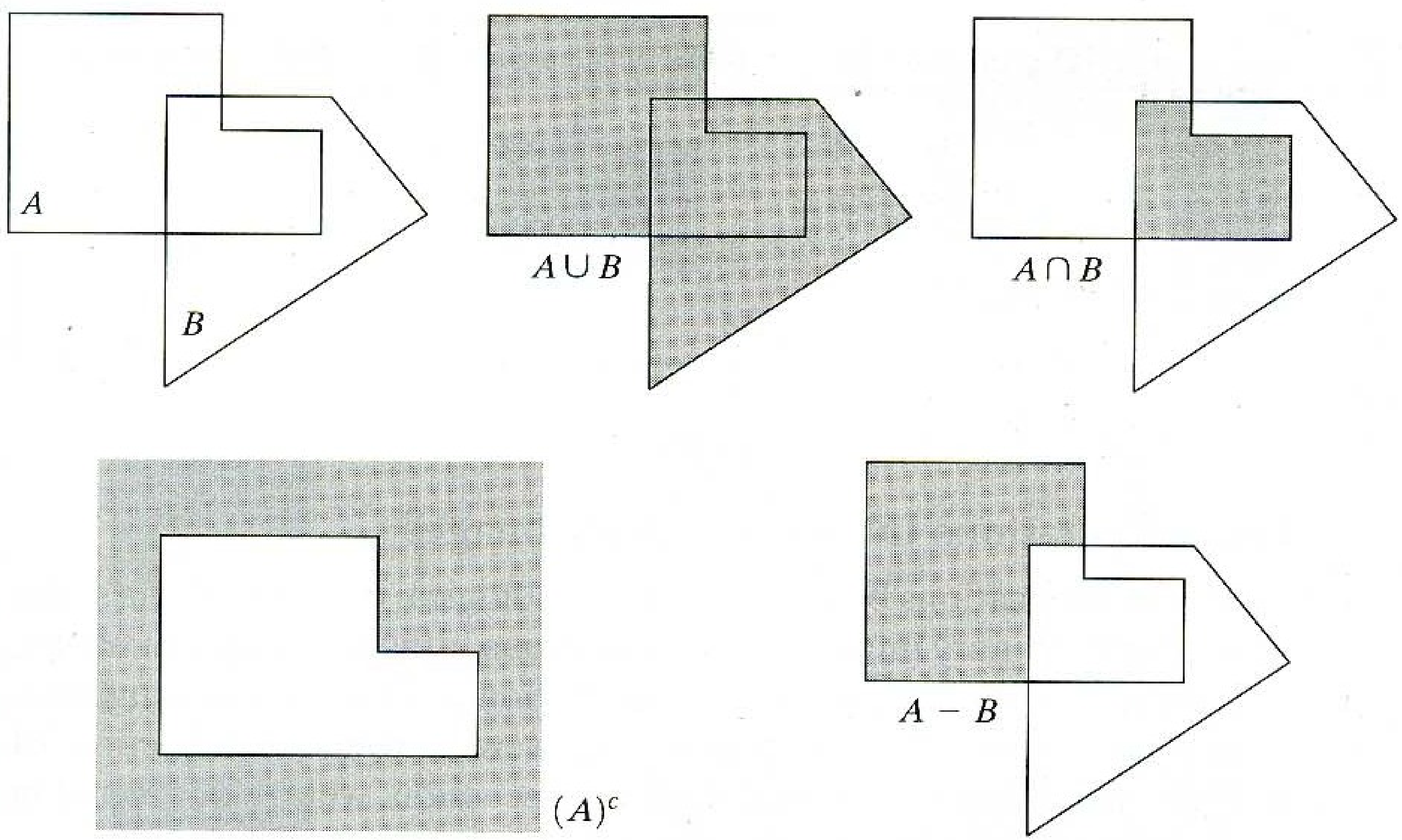
Basic Concepts from Set Theory

- A set like B in Z^2 with elements $a = (a_1, a_2)$ is defined as

$$B = \{w \mid w = (a_1, a_2) \ \ for \ \ a_1, a_2 \in Z\}$$

- Union, Intersection, Set Difference, and Complement are defined as shown in the figure.

Basic Concepts from Set Theory



Reflection and Translation

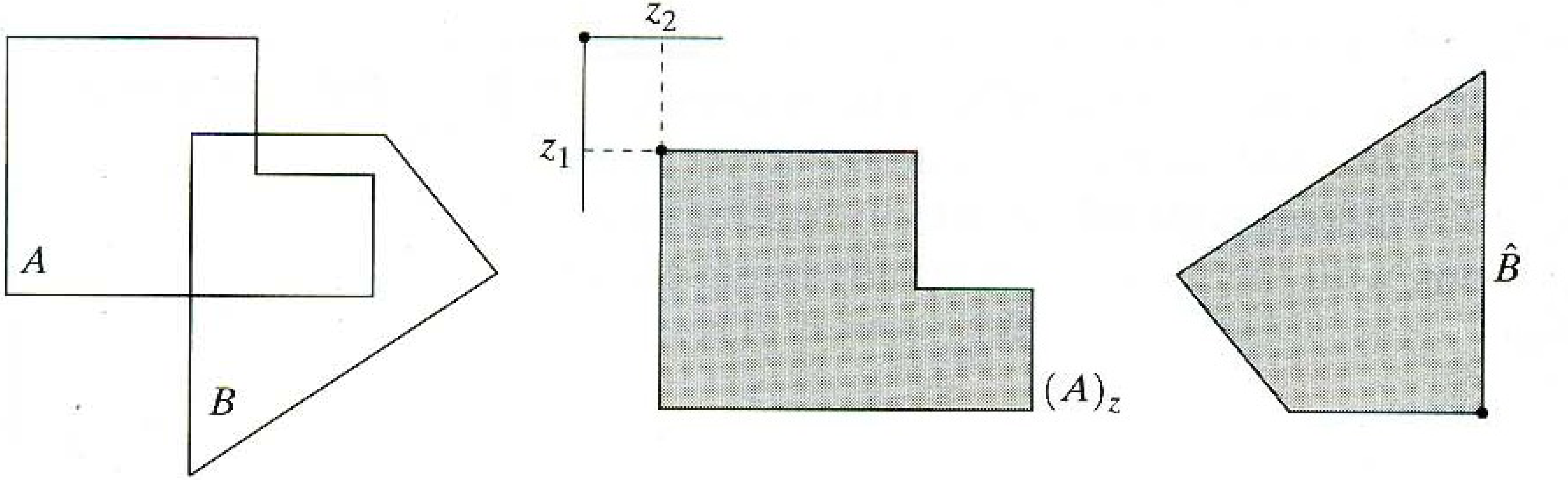
- Reflection of a set B is defined as

$$\hat{B} = \{w \mid w = -b, \quad \text{for} \quad b \in B\}$$

- Translation of a set A by a point z = (z_1, z_2)is defined as:

$$(A)_z = \{c \mid c = a + z, \quad \text{for} \quad a \in A\}$$

Reflection and Translation

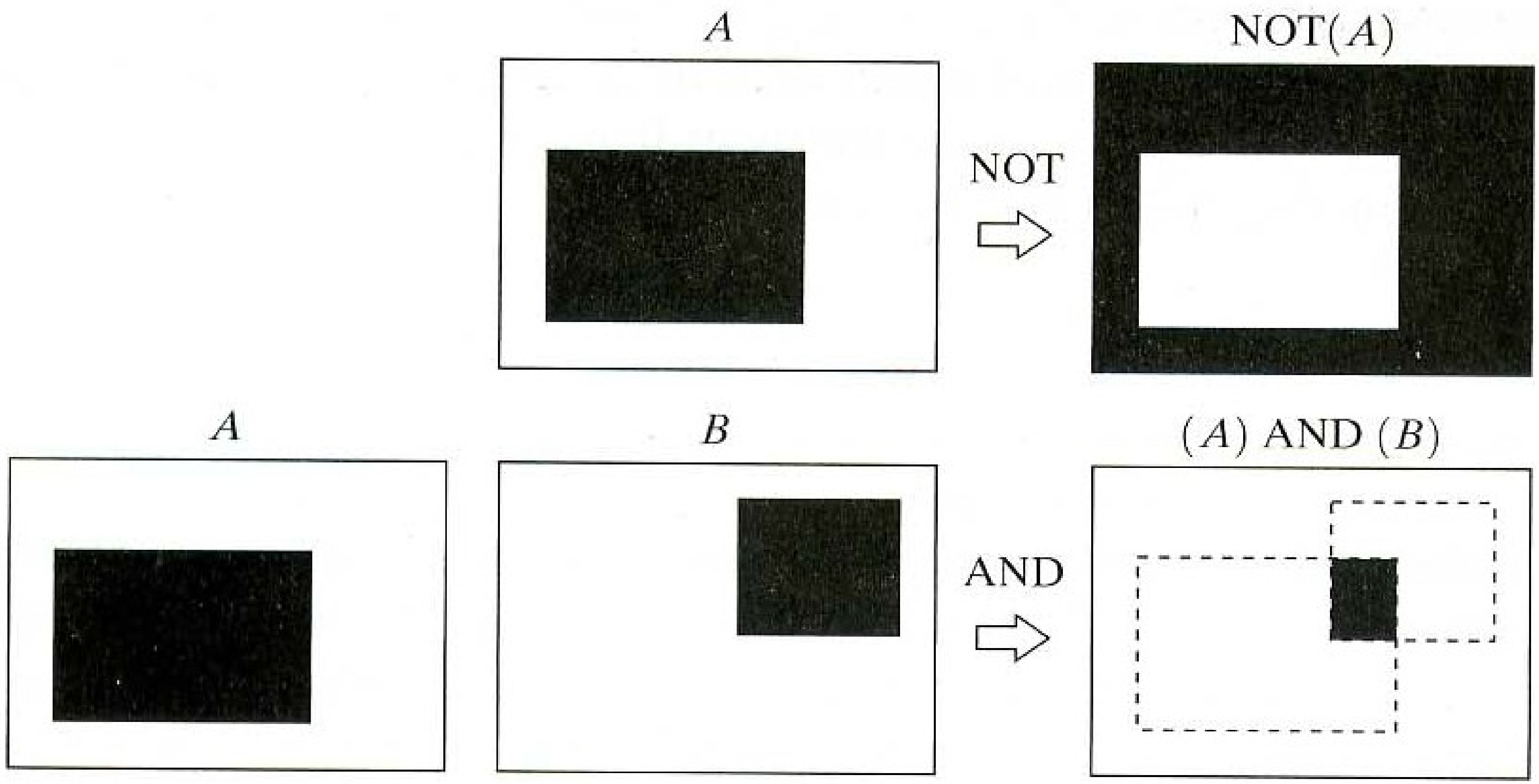


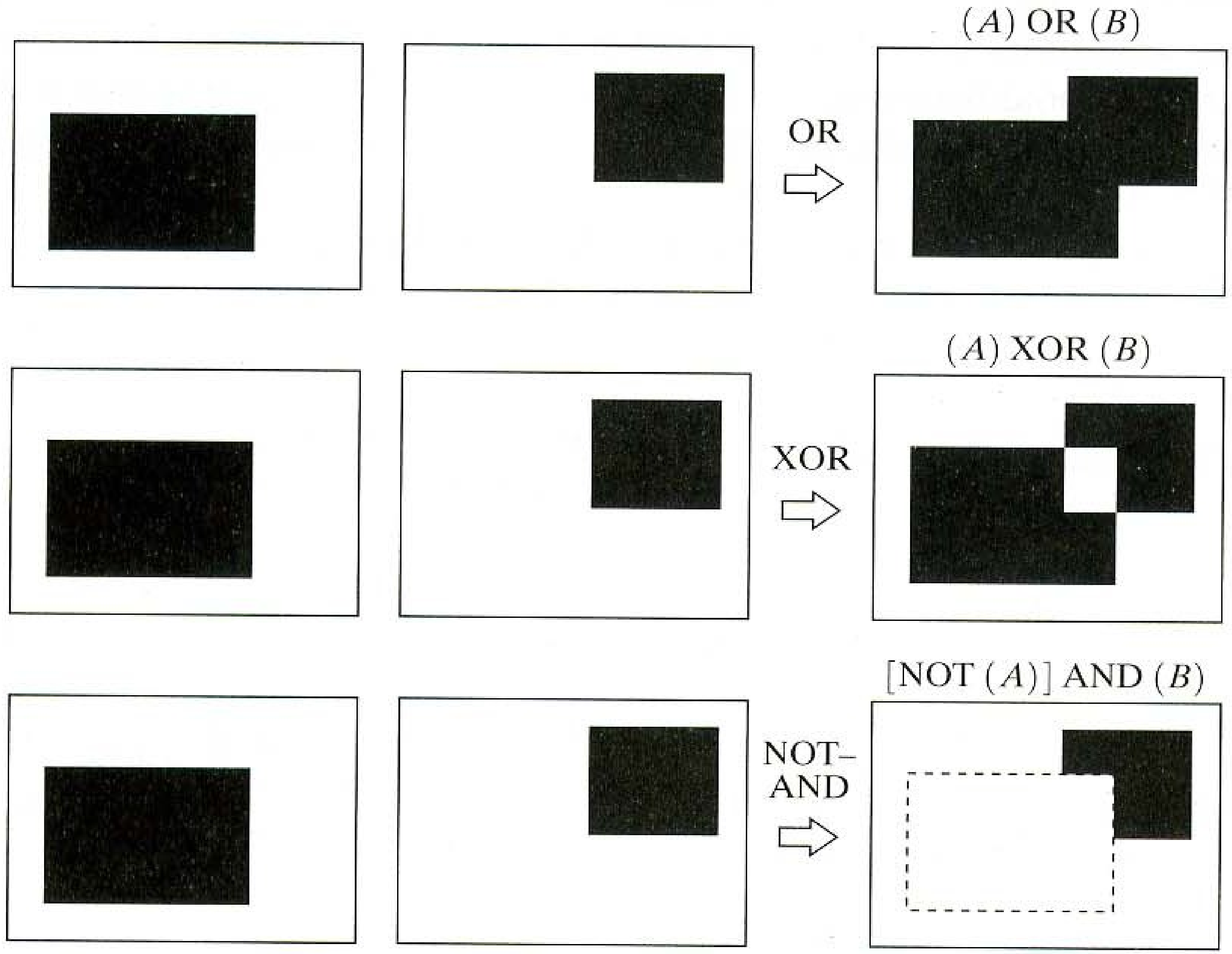
Logical Operation on Binary Images

- The properties of logical operators are given below

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Logical Operations on Binary Images




(A) OR (B)
OR
(A) XOR (B)
XOR
[NOT (A)] AND (B)
NOT–
AND

Dilation and Erosion

- Dilation and Erosion are two basic operations in morphological processing.
- Dilation of a set A in Z^2 by a set B in Z^2 is denoted by $A \oplus B$ and given by

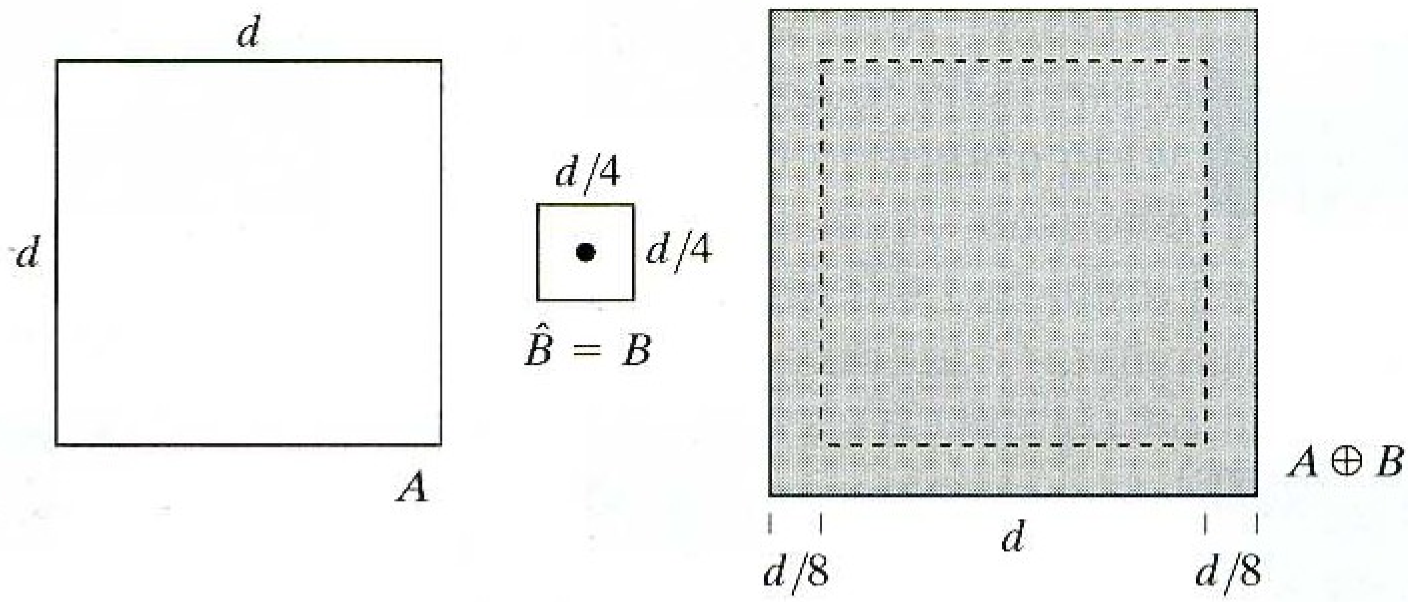
$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \varnothing \}$$

Dilation

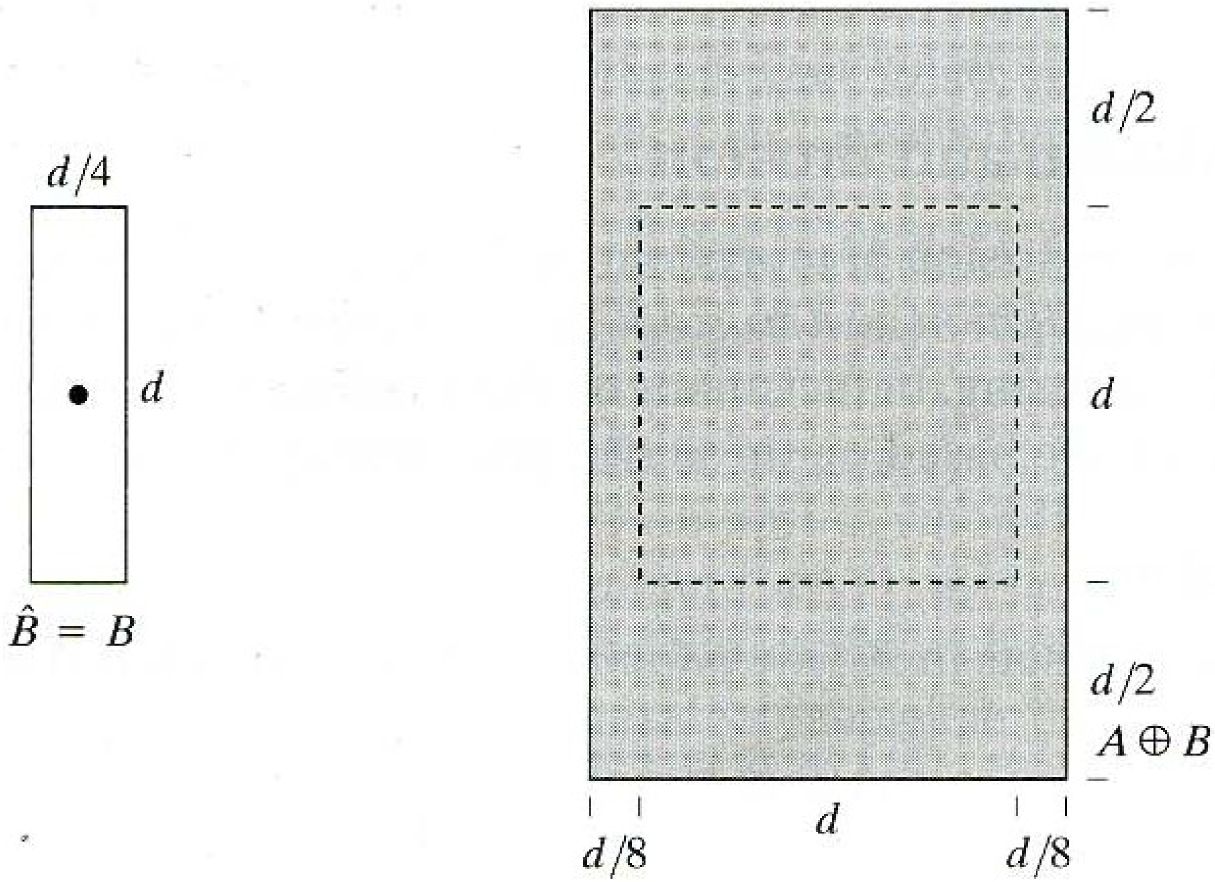
- The dilation of A by B is the set of all displacements such that A and $\hat{B}$ overlap with at least one point

- B is called structuring element

Dilation



Dilation



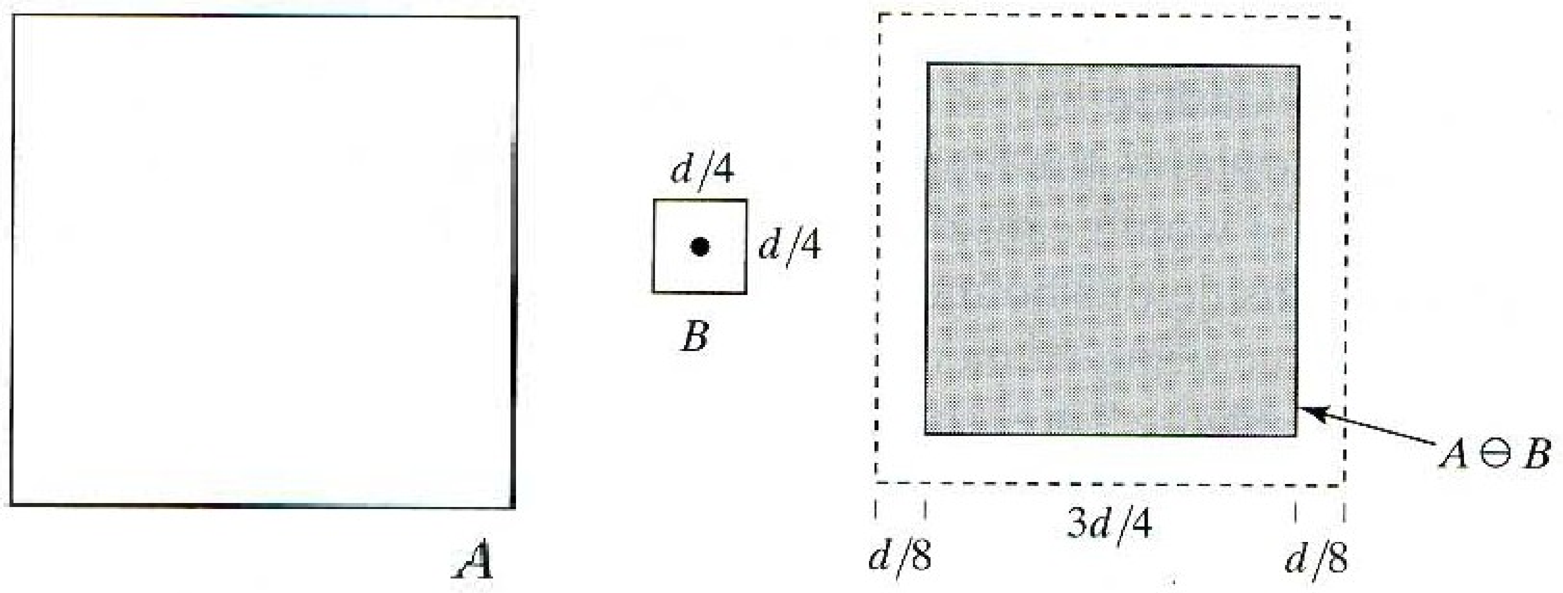
Erosion

- Erosion of a set A in Z^2 by a set B in Z^2 is denoted by $A \ominus B$ and given by:

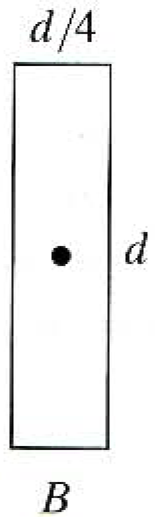
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

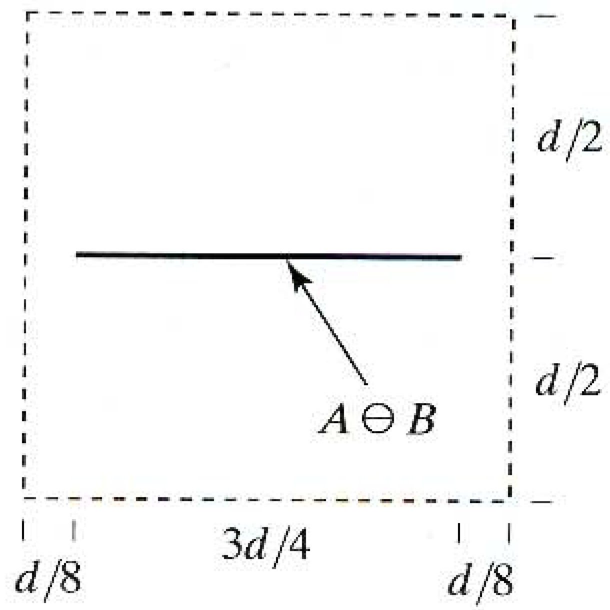
Erosion of A be B is the set of all points z such that B translated by z is contained in A

Erosion

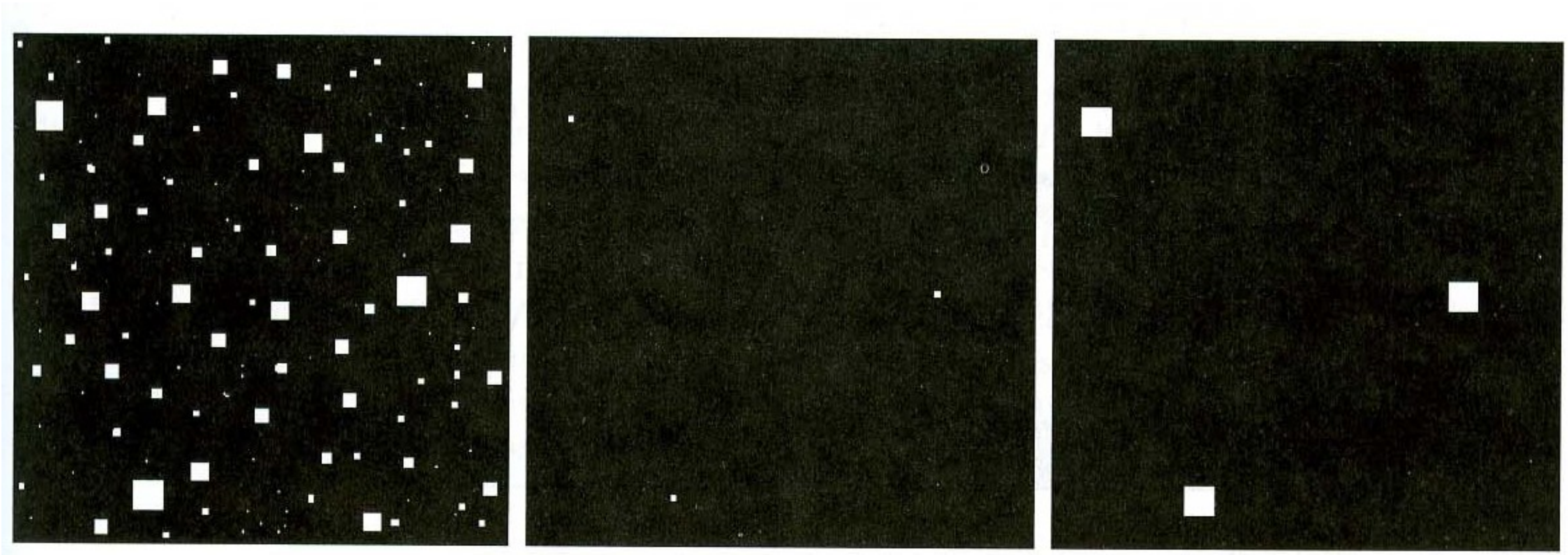


Erosion





Example: Applying Erosion and then Dilation



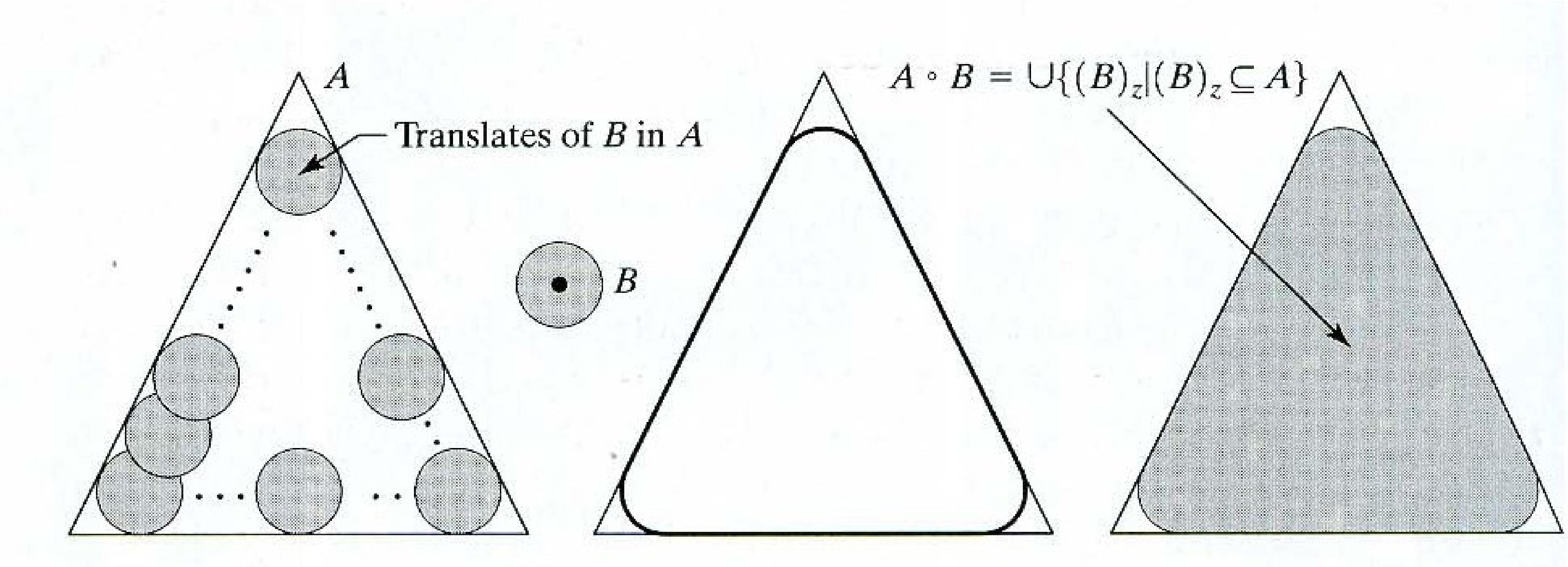
Opening

- Opening smoothes the outer contours, breaks narrow connections, and eliminates small protrusions.

- Opening is defined as :

$$A \circ B = (A \ominus B) \oplus B$$

Opening

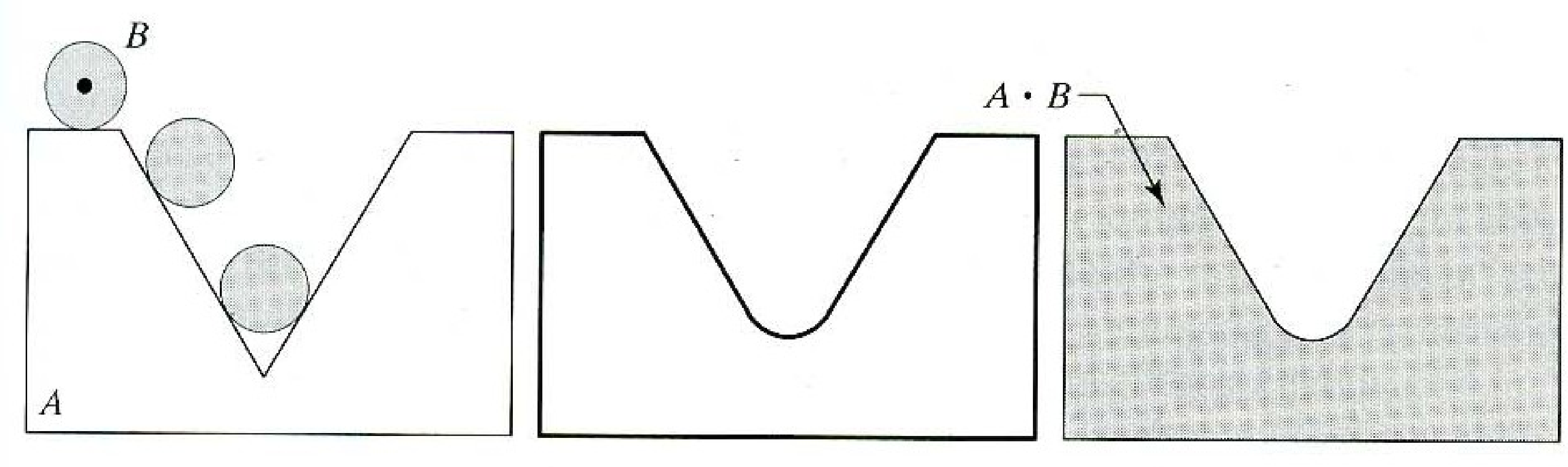


Closing

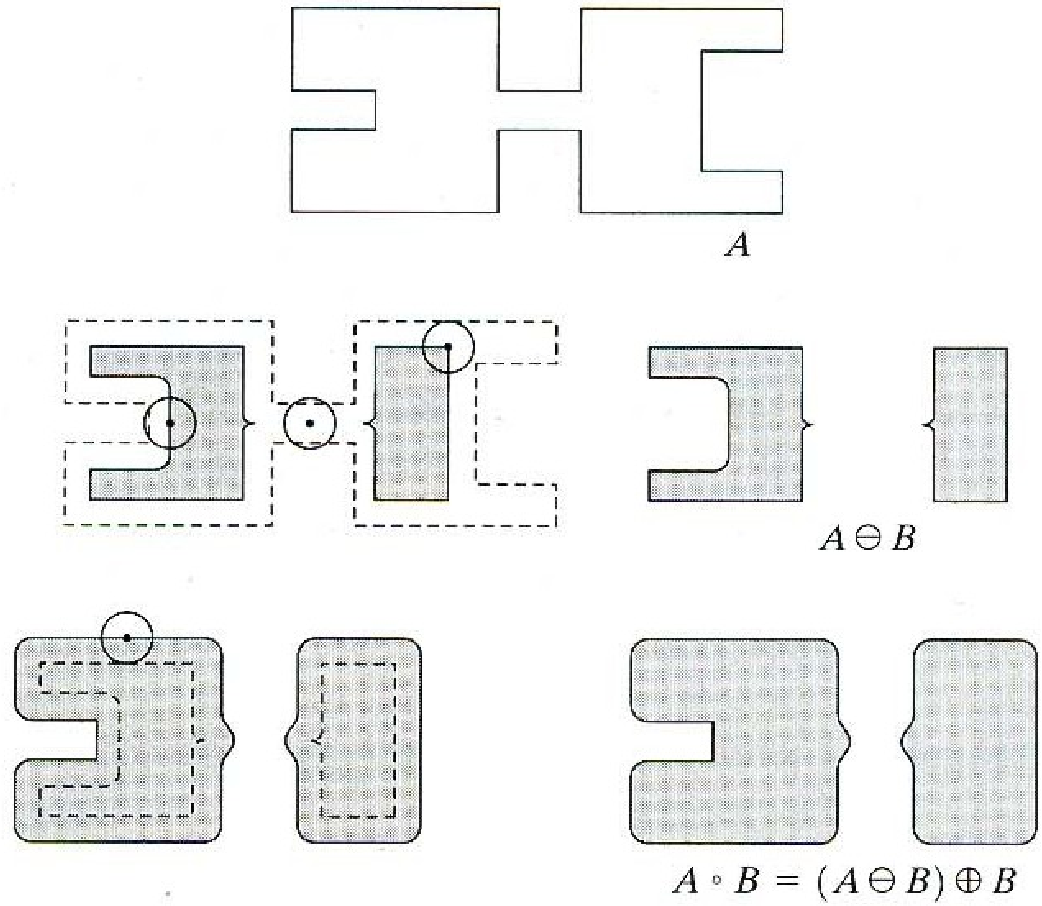
- Closing smoothes the object contour, fuses narrow connections, eliminates small holes and gaps.

$$A \bullet B = (A \oplus B) \ominus B$$

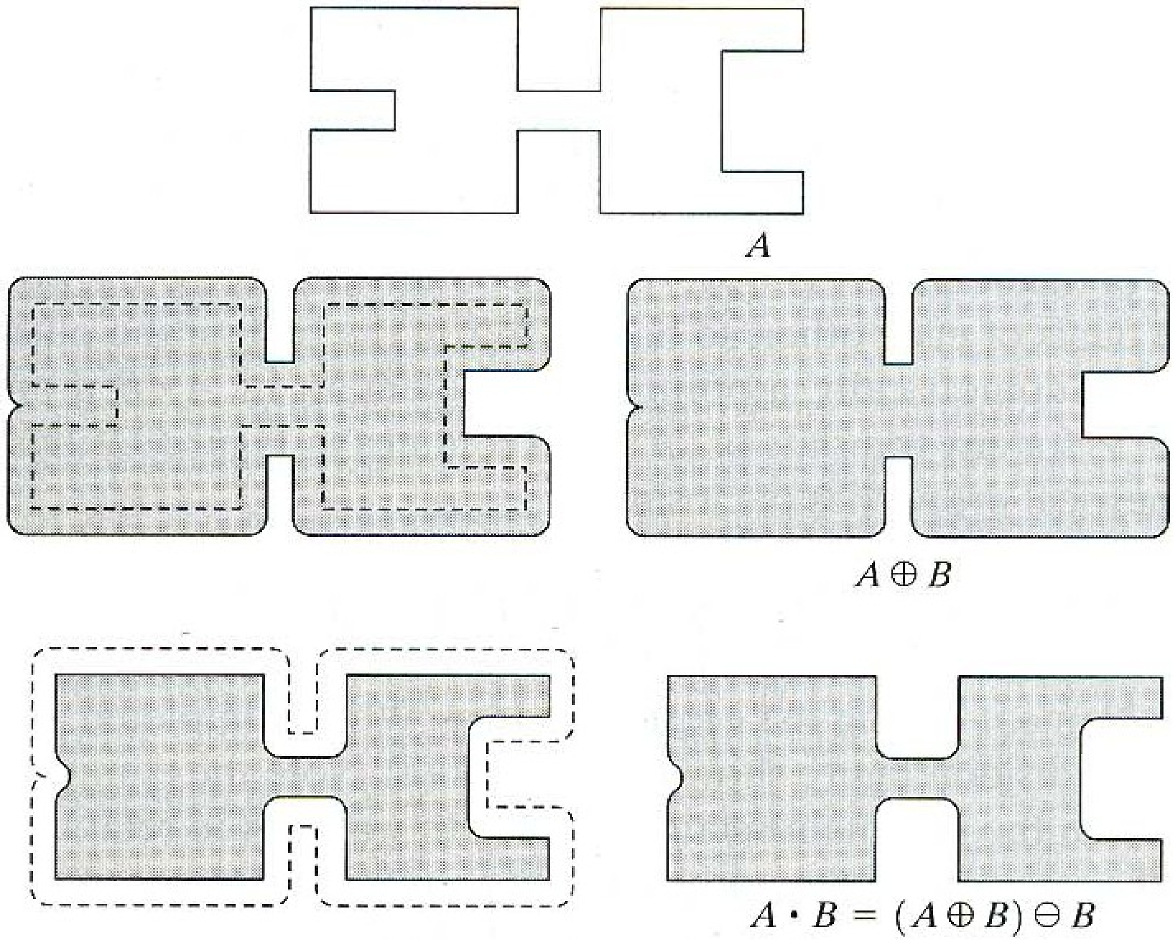
Closing



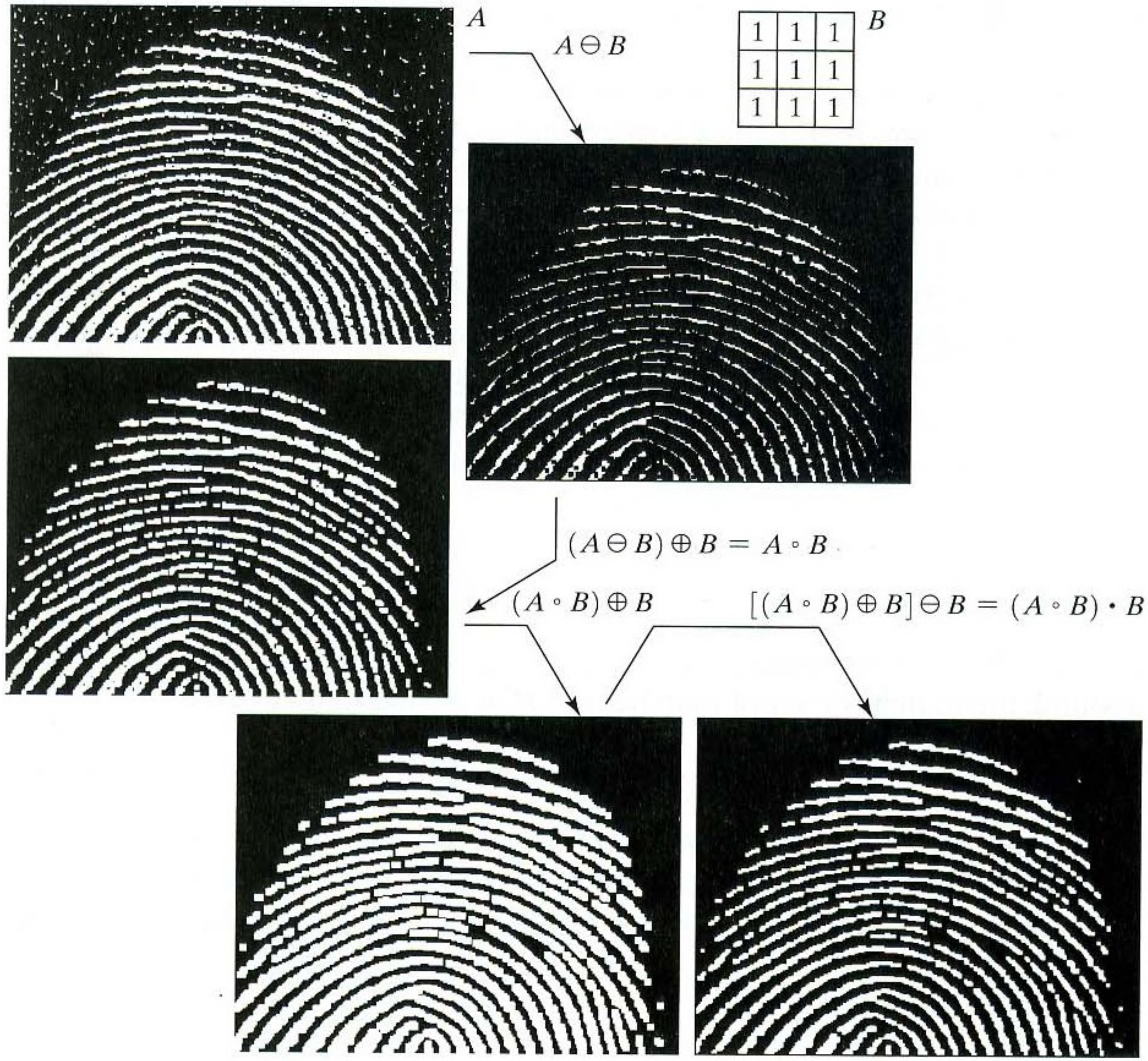
Opening



Closing



Example



Hit-or-Miss

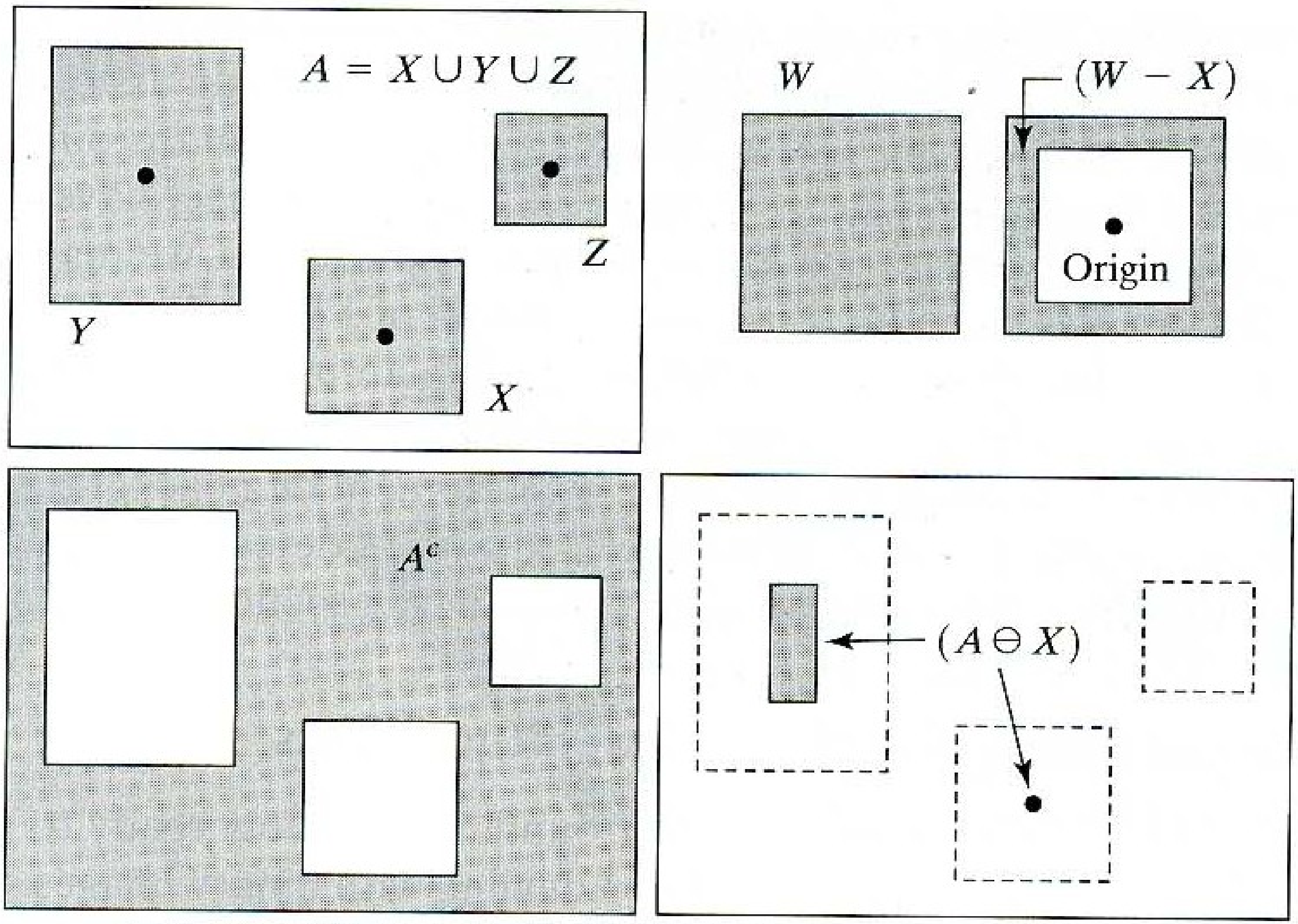
- Hit-or-Miss is used for shape detection.

- Hit-or-Miss is defined as:

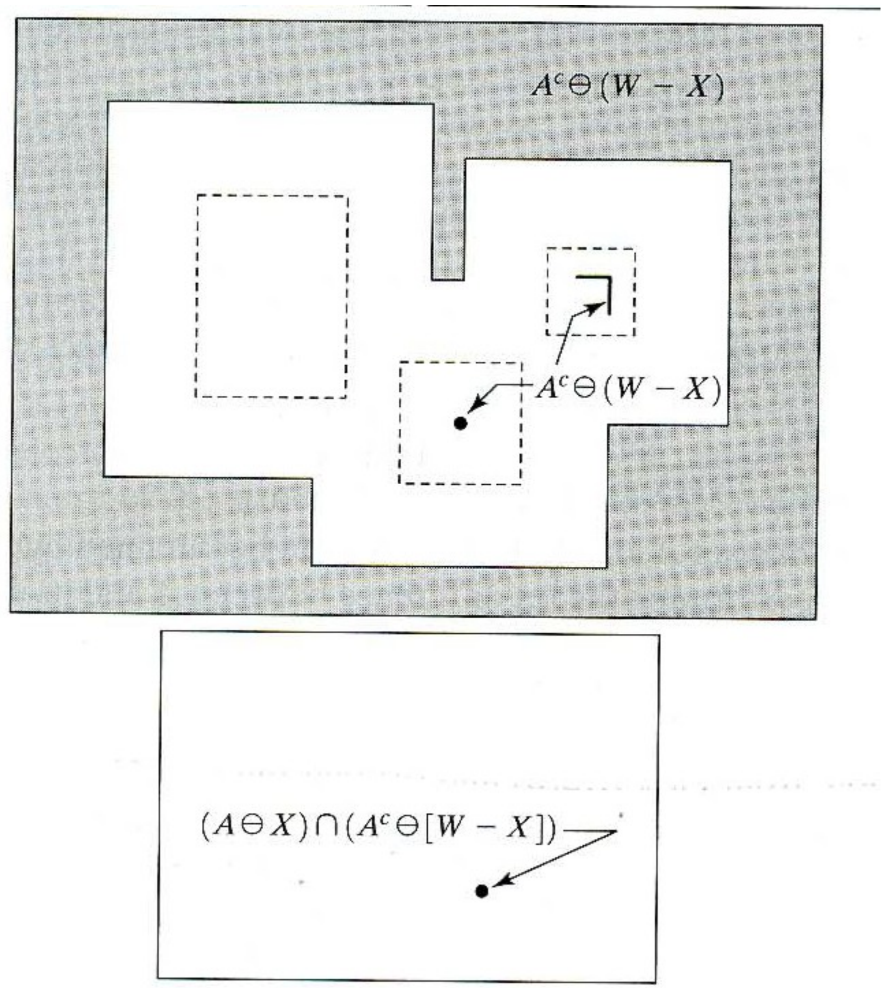
$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

where B = (B1, B2), B1 is the shape we are looking for, and B2 is its background given as W-B1 (W is a small window containing B1)

Hit-or-Miss

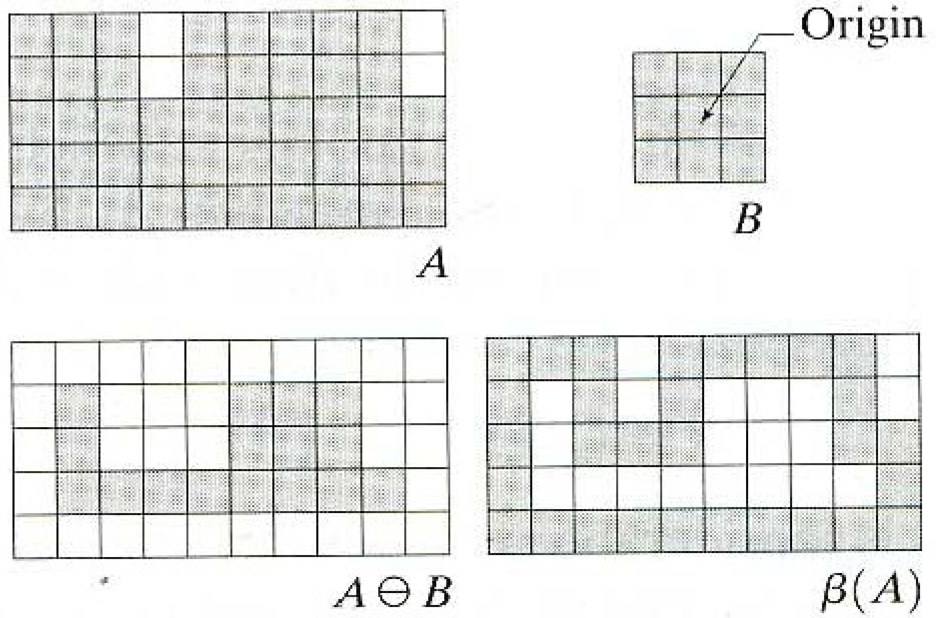


Hit-or-Miss

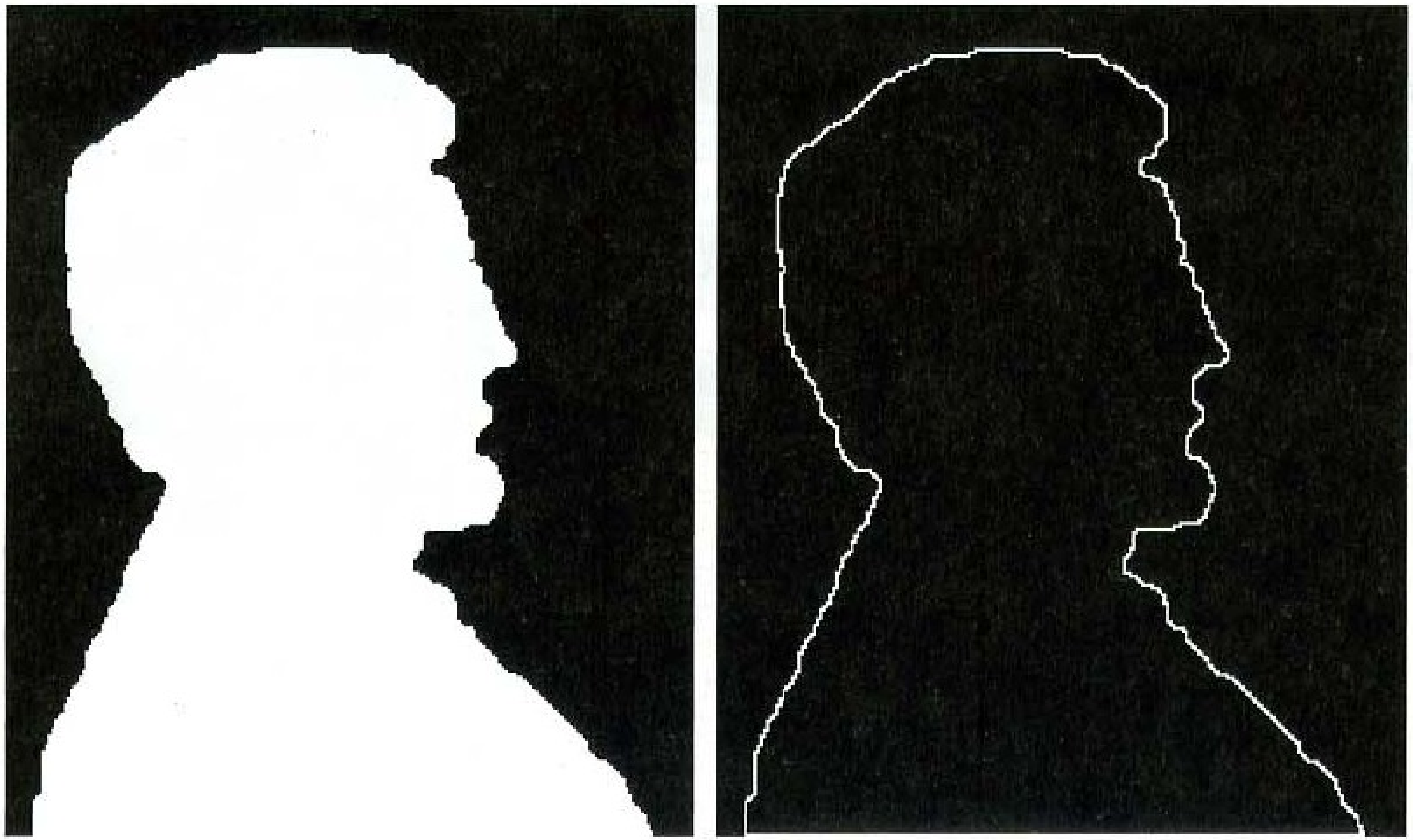


Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$

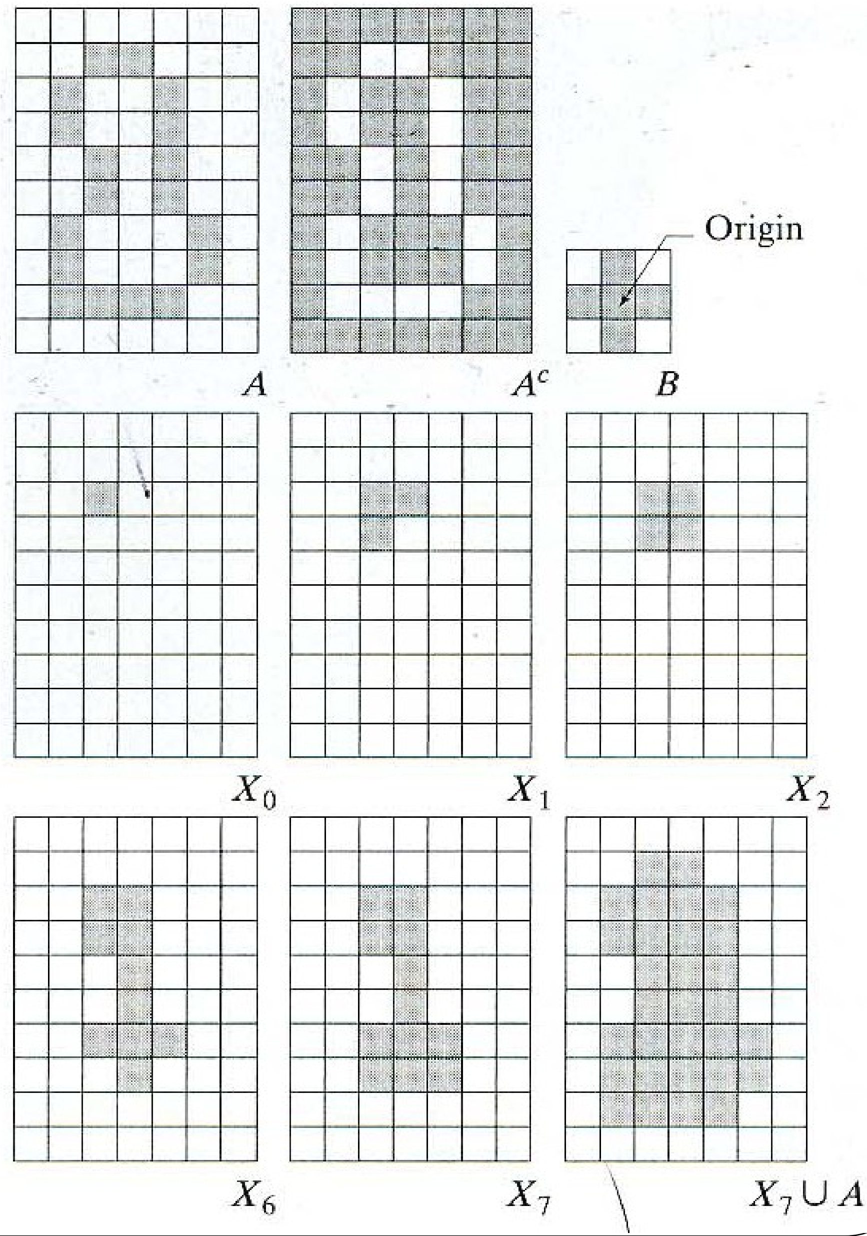


Boundary Extraction Example

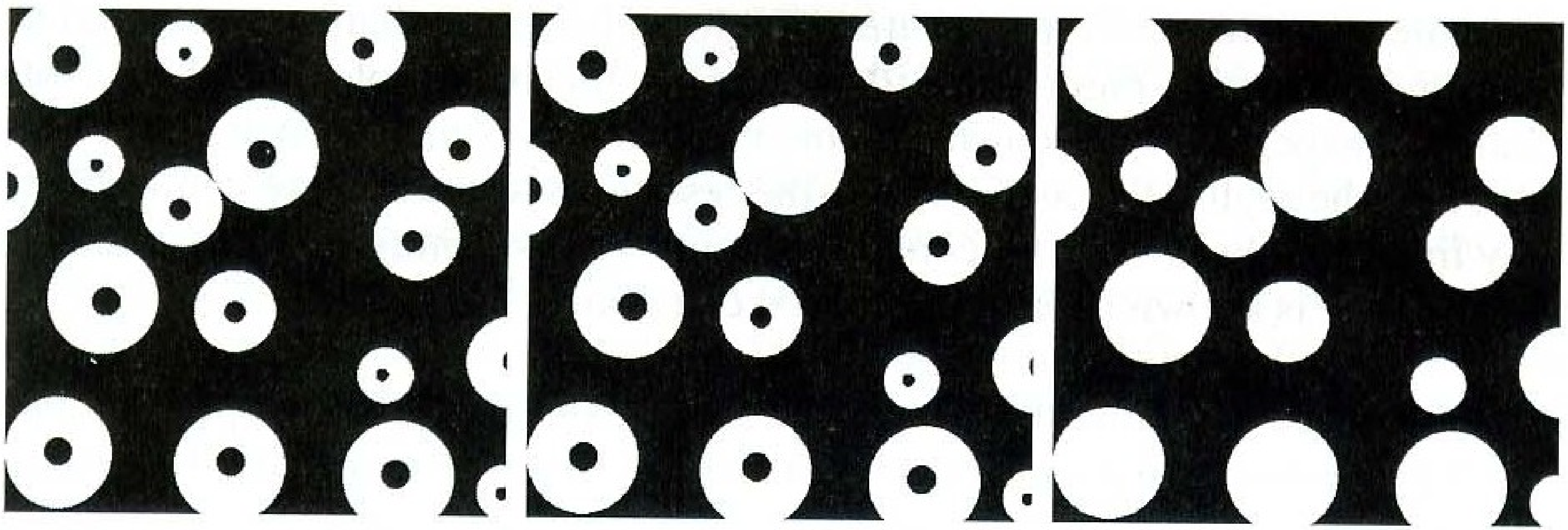


Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \ldots$$



Region Filling



More Operations

- Connected Component Extraction
- Convex Hull
- Thinning
- Thickening
- Skeleton
- Pruning
- Extension to gray level images

Questions?

Assignment

- Assume an imaging system is used to inspect the accuracy of washers produced in a manufacturing plant. Assume the image of an error free washer is provided. Develop an algorithm using morphological operators for this problem. You may download test images from the course web page.