Digital Image Processing

Active Contours

Topics

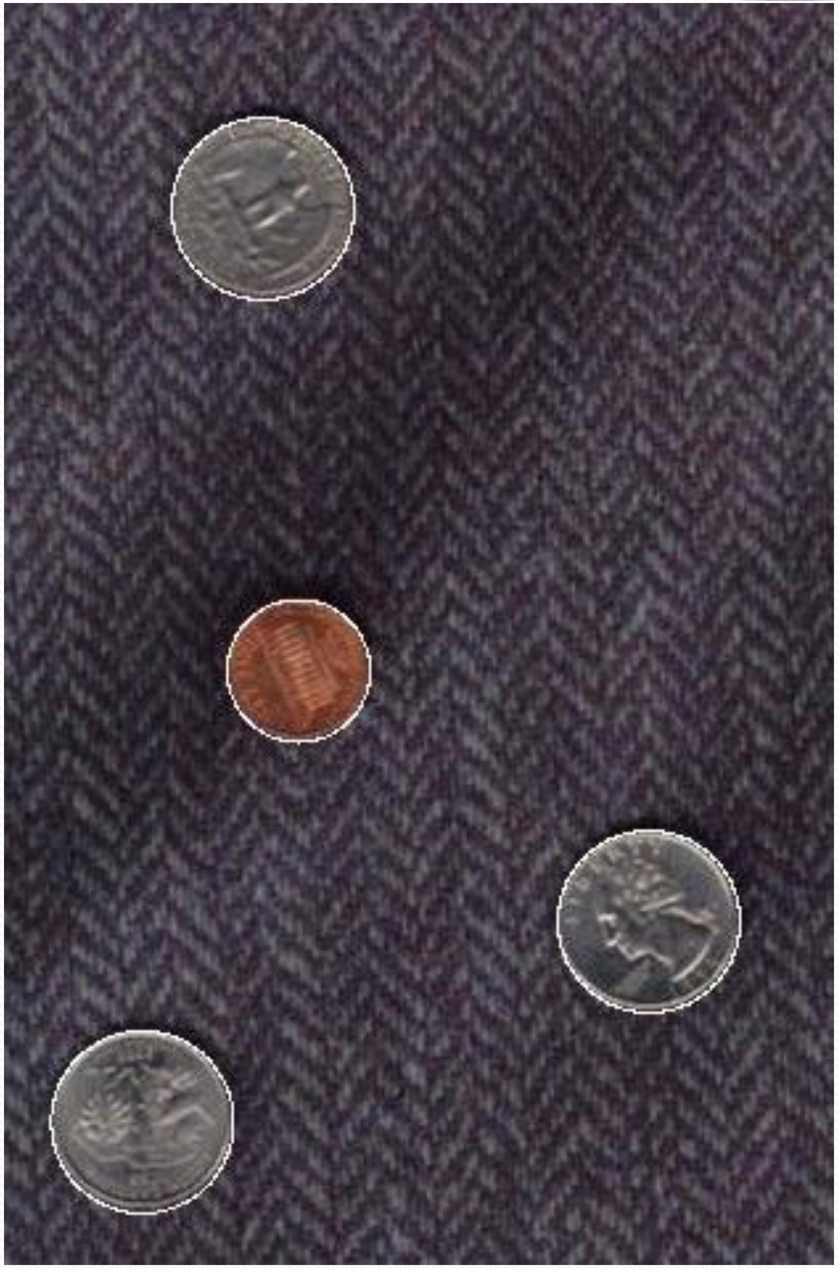
- Motivation
  - Edge and Boundary
  - Boundary Detection
- Active Contours
  - Representation
  - Energy Function
  - Minimizing the Energy Function

Motivation

- How can we find the boundaries of an object?
- Edge pixels can be used to find boundaries but quite often boundaries of interest are fragmented, and we have extra "cluttered" edge points.

Boundaries

- Detecting boundaries requires grouping of the pixels
- Given a model of the object, we can overcome some of the missing and noisy edges using **fitting** techniques.
- With voting methods like the **Hough Transform**, detected points vote on possible model parameters.



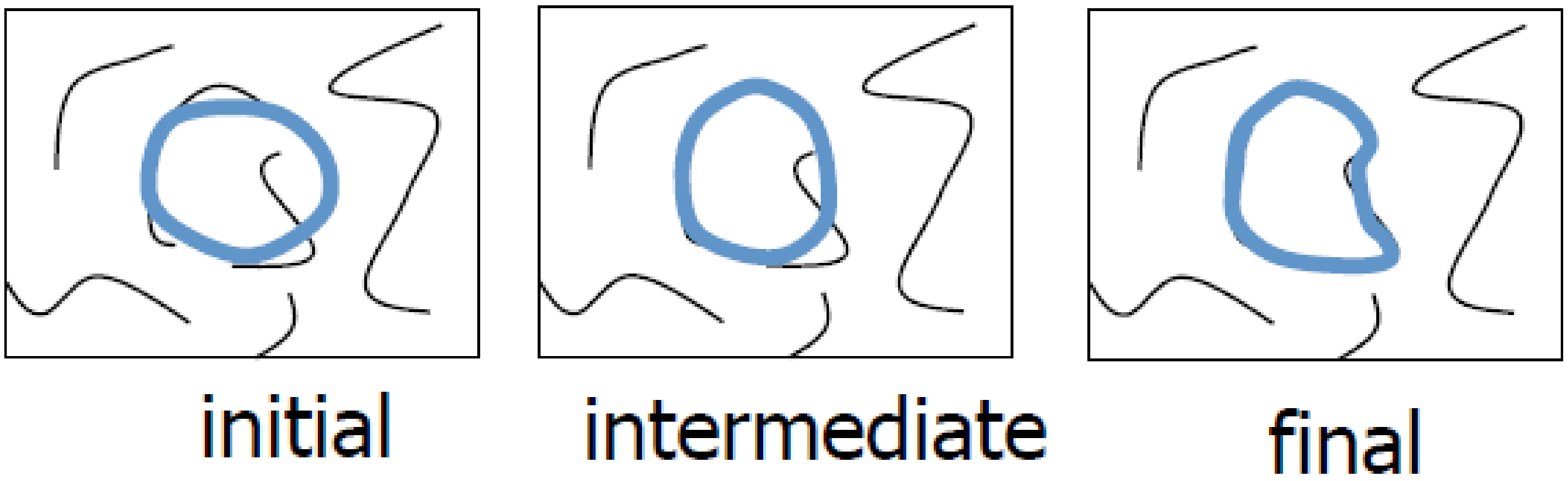
Active Contours

- **Given**: initial contour (model) near desired object
- **Goal**: evolve the contour to fit exact object boundary

- **Main idea**: elastic band is iteratively adjusted so as to be near image positions with high gradients, **and** satisfy shape "preferences"


Active Contours vs. Hough Transform

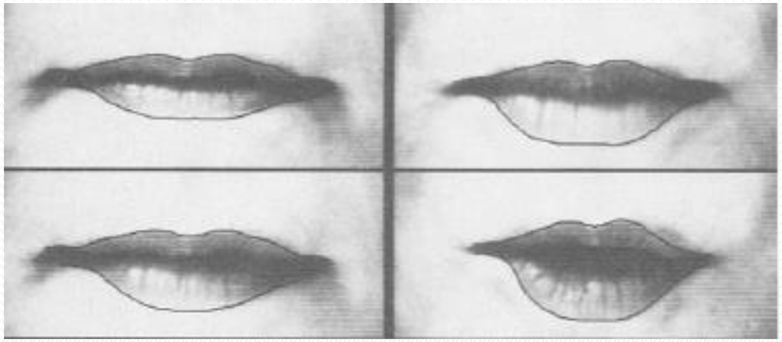
- Like generalized Hough transform, useful for shape fitting; but:
  - In Hough Transform we have
    - Rigid model shape
    - Single voting pass can detect multiple instances
- Active contours
  - Prior on shape types,
  - but shape iteratively adjusted (*deforms*)
  - Requires initialization nearby,
  - One optimization “pass” to fit a single contour


initial
intermediate
final

Deformable Shapes

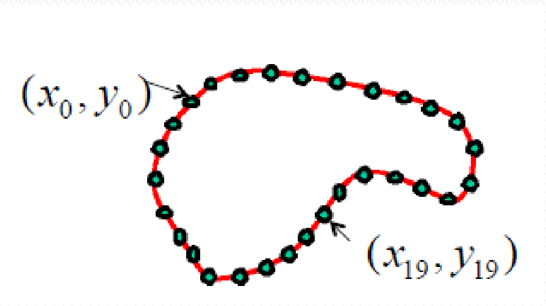
- Some objects have similar basic form but some variety in the contour shape

- Non-rigid, deformable objects can change their shape over time, e.g. lips, hands…

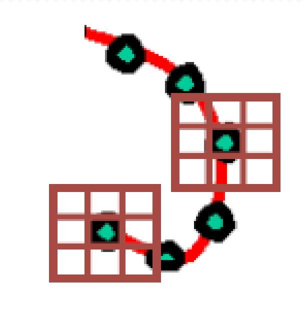


Contour Representation

- We'll consider a discrete representation of the contour, consisting of a list of 2d point positions ("vertices").



- At each iteration, we'll have the option to move each vertex to another nearby location ("state").



Fitting Contour to Image

- How should we adjust the current contour to form the new contour at each iteration?
  - Define a cost function ("energy" function) that says how good a candidate configuration is.
  - Seek next configuration that minimizes that cost function.

Energy (Cost) Function

- The total energy (cost) of the current contour is defined as:

$$E_{total} = E_{internal} + E_{external}$$

- **Internal** energy: encourage *prior* shape preferences: e.g.,
  - smoothness, elasticity, particular known shape.
- **External** energy ("image" energy): encourage contour to fit on places where image structures exist, e.g., edges
- A good fit between the current active contour and the target shape in the image will yield a **low** value for this cost function.

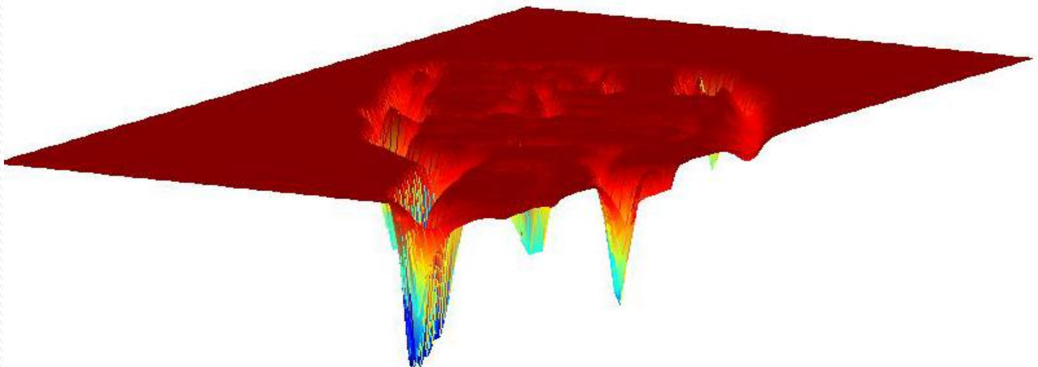
External Energy

- **External energy: intuition**
- Measure how well the curve matches the image data
- “Attracts” the curve toward different image features such as edges, lines, texture gradient, etc.

External Energy

- **External image energy**
- Magnitude of gradient
  - (Magnitude of gradient) = $-(G_x(I)^2 + G_y(I)^2)$

- How do edges affect “snap” of a rubber band?
- Think of external energy from image as gravitational pull towards areas of high contrast



External Image Energy

- External energy is defined as the Gradient of the image.
- External energy at a point on the curve is:

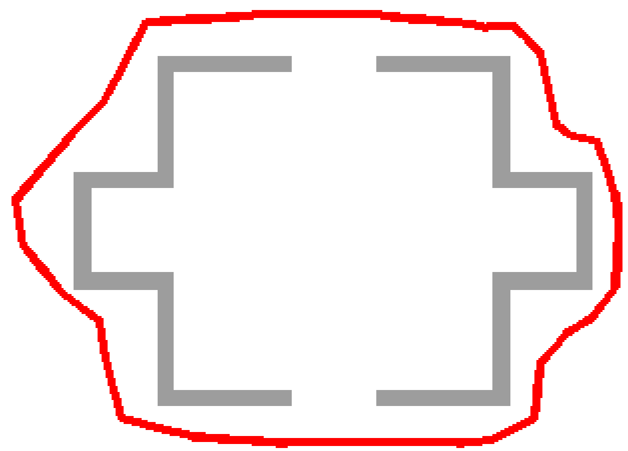
$$E_{External}(v) = -(G_x(v)^2 + G_y(v)^2)$$

- External energy for the whole curve:

$$E_{External} = -\sum(G_x(v_i)^2 + G_y(v_i)^2)$$

Internal Energy

- *A priori*, we want to favor **smooth** shapes, contours with **low curvature**, contours similar to a **known shape**, etc. to balance what is actually observed (i.e., in the gradient image).



Internal Energy

- For a *continuous* curve, a common internal energy term is the “bending energy”.
- At some point *v(s)* on the curve, this is:

$$E_{internal}(v(s)) = \alpha \left| \frac{dv}{ds} \right|^2 + \beta \left| \frac{d^2 v}{d^2 s} \right|^2$$

Internal Energy

- In discrete representation we have:

$$v_i = (x_i, y_i) \qquad i = 0 \ldots n-1$$

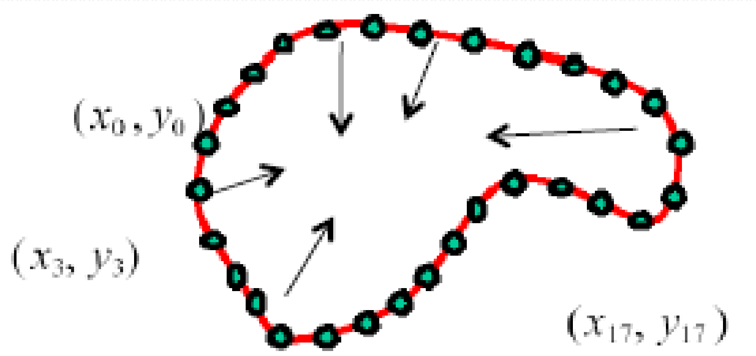
$$\frac{dv}{ds} \approx v_{i+1} - v_i \qquad \frac{d^2v}{ds^2} \approx (v_{i+1} - v_i) - (v_i - v_{i-1}) = v_{i+1} - 2v_i + v_{i-1}$$

- The internal energy for the whole curve is:

$$E_{internal} = \sum_{i=0}^{n-1} \alpha \left\| v_{i+1} - v_i \right\|^2 + \beta \left\| v_{i+1} - 2v_i + v_{i-1} \right\|^2$$

Penalize Elasticity

- Current elastic energy definition uses a discrete estimate of the derivative:



$$E_{elastic} = \sum_{i=0}^{n-1} \alpha \left\| v_{i+1} - v_i \right\|^2$$

$$= \alpha \cdot \sum_{i=0}^{n-1} (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2$$

Penalize Elasticity

- Instead we prefer using:

$$= \quad \alpha \cdot \sum_{i=0}^{n-1} \left((x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 - \overline{d} \right)^2$$

- Where d is the average distance between the pairs of curve points updated at each iteration

Total energy: Function of the Weights

$$E_{total} = E_{internal} + \gamma E_{external}$$

$$E_{external} = -\sum_{i=0}^{n-1} |G_x(x_i, y_i)|^2 + |G_y(x_i, y_i)|^2$$

$$E_{internal} = \sum_{i=0}^{n-1} \alpha \left(\bar{d} - \|v_{i+1} - v_i\|\right)^2 + \beta \|v_{i+1} - 2v_i + v_{i-1}\|^2$$

Total energy: Function of the Weights

- For example α weight controls the penalty for internal elasticity

Summary

- **Active contours: pros and cons**
- Pros:
  - Useful to track and fit non-rigid shapes
  - Contour remains connected
  - Possible to fill in "subjective" contours
  - Flexibility in how energy function is defined, weighted.
- Cons:
  - Must have decent initialization near true boundary, may get stuck in local minimum
  - Parameters of energy function must be set well based on prior information

Questions?